DEVELOPMENT TEAM

Project Leader	Jim Dale	
Software Design	Michel Pitre Rob McLeish	
Project and Support Team	Rob McLeish Elizabeth Klassen Vic D'Amico Rhéal Dumont Bob Armstrong	Karen Brousseau Alex Belloni Jacinthe Perras Christine Chapman

WARRANTY

NECTAR Foundation warrants the enclosed diskettes on which the program is recorded to be free of any defects in materials and faulty workmanship under normal use for a period of 30 days after purchase. In the event you receive a defective diskette, you may receive a replacement diskette by sending the defective diskette, postage prepaid to:

NECTAR Foundation
570 West Hunt Club Road
Nepean, Ontario K2E 3R4

Updates to this program may become available. Proof of purchase may be required.

RESTRICTIONS
This program is sold with a single-user license for use on one computer. Lab Packs and Network/Site License versions are available.
Call 1-613-224-3031 for further information.

Windows - ISBN # 1-55112-049-6 Macintosh - ISBN # 1-55112-070-4

TABLE OF CONTENTS

INTRODUCTION TO VIRTUAL TILES

Virtual Tiles is a software program which gives students of all ages a concrete model for operations with integers and polynomials. It is a computerized version of the algebra tiles manipulative used by many teachers. The evaluation of an expression, combining like terms, solving equations, the distributive property, multiplication of polynomials, division of polynomials, and factoring, all have concrete models using tiles which are extremely effective. *Virtual Tiles* allows a student to construct her/his own meaning for many complicated concepts. Students can then factor expressions with *Virtual Tiles* that very few students in high school could do using any other technique. This software provides all the benefits of working with algebra tiles or blocks with many significant advantages.

- much faster and precise than moving tiles around by hand
- can automate many of the features (including filling in the gaps with tiles, filling in the guide tiles, labeling the dimensions, shows the equivalence as a multiplication, division or factorization, etc.), so students can avoid practicing steps they already know how to do well.
- Can use as a symbolic calculator.
- much larger and more complicated examples can be dealt with in this environment than with physical manipulatives since Virtual Tiles are in two variables (x^2, y^2, xy, x, y, and 1) and there are virtually an unlimited number of expressions. For example, the polynomial $6x^2 - 2y^2 - xy - 4x + 5y - 2$ can be easily factored with Virtual Tiles.
- turning over tiles is as simple as a double click of the mouse, so the concept of multiplying by a negative is easily demonstrated.
- classroom management is made simpler. Virtual tiles are automatically distributed to students as soon as they boot-up their computer, cannot be destroyed or lost, and don't have to be collected afterwards.

This product has been tested successfully by children as young as 8 years old. They have been able to master operations with polynomials in a fun, interactive two-dimensional environment. It has also been used by students from grades 6 - 12 to introduce and reinforce algebraic concepts.

Teachers can also use *Virtual Tiles* as a demonstration device using a projection tablet. Many concepts can be introduced clearly and effectively in a large group setting using *Virtual Tiles*.

Virtual Tiles is available in both Windows and Macintosh formats in English, French and Spanish.

INSTALLATION

Windows 3.x

1. Insert the *Virtual Tiles* disk into your 3.5" disk drive.
2. From the Windows **PROGRAM MANAGER** click on **FILE** and then select **RUN**.
3. Type **A:install** or **B:install**, depending on your drive designation.
4. Follow the instructions that appear on the screen. A *Virtual Tiles* directory will be created.
5. To run *Virtual Tiles*, click the icon *Virtual Tiles* in the Program Manager.
6. If you are installing *Virtual Tiles* on a network, designate the *Virtual Tiles* file as ROS (read only, shareable).

Windows 95

1. Insert the *Virtual Tiles* disk into your 3.5" disk drive.
2. From the **START** menu select **RUN**.
3. Type **A:install** or **B:install**, depending on your drive designation.
4. Follow the instructions that appear on the screen. A *Virtual Tiles* directory will be created.
5. To run *Virtual Tiles*, click the icon *Virtual Tiles* in the Program Manager.
6. If you are installing *Virtual Tiles* on a network, designate the *Virtual Tiles* file as ROS (read only, shareable).

Macintosh

1. Insert the *Virtual Tiles* disk into your floppy drive.
2. Double click on the disk icon and you will see the file called *Virtual Tiles*. Drag this file to the location of your choice on your hard drive.
3. To run *Virtual Tiles,* click the application entitled *Virtual Tiles*.

REGISTRATION AND TECHNICAL SUPPORT

Please fill out and mail the enclosed Registration Card. Doing so will ensure that you are eligible for technical support, and that you will be notified of updates and other information.

The NECTAR Foundation is committed to our customers. Technical support is provided at no charge to our registered customers.

Before calling, please:

- have a clear idea of what you want to ask (write down your questions)
- be prepared to duplicate the problem on your computer (write down the sequence or combination of keystrokes, dialogue choices, or menu commands that caused the problem)
- note any error messages, strange symbols, or any unusual occurrence.

Have the following information available when you call and be sure to specifically ask for Technical Support for *Virtual Tiles*.

• Your Name	
• Your School Name	
• Your Phone Number	
• Your Computer Model	(e.g. Macintosh LC 575 or IBM 485)
• Your Printer Model	(e.g. StyleWriter Pro or HP DeskJet 500)
• Windows Version	(e.g. MS Windows 3.1)
• Macintosh System Version	(e.g. System 7.1)
• Free RAM available to your system	
• *Virtual Tiles* Version	(e.g. 1.0)

You can reach NECTAR Foundation Technical Support via phone or FAX:

Support Hours	8:00 AM - 4:00 PM, Eastern Standard Time
Technical Support Phone Number	
within Canada and the United States	1-800-387-1964 or 1-613-224-3031
Technical Support FAX Numbers	1-613-224-1946

NOTE: *Virtual Tiles* was developed in Canada using the Canadian spelling for words such as colour, and metre.

LEARNING OUTCOMES

The student will be able to use *Virtual Tiles* to:

1. Manipulate tiles in:
   - the Addition Environment
   - the Multiplication Environment
   - the Equations Environment

2. Model integers, variables, algebraic expressions and linear equations and corresponding operations.

3. Add, subtract, multiply and divide integers.

4. Simplify expressions by combining like terms.

5. Evaluate an expression.

6. Solve linear equations in one variable.

7. Isolate one variable in linear equations containing two variables.

8. Multiply polynomials.

9. Divide polynomials (includes expressions of the form $ax^2 + bxy + cy^2 + dx + ey + f$ divided by expressions of the form $mx + ny + p$).

10. Complete the square using expressions of the form $ax^2 + bxy + cy^2 + dx + ey + f$.

11. Factor polynomials of the form $ax^2 + bxy + cy^2 + dx + ey + f$ (includes common factors, difference of squares, perfect squares, all forms of trinomials factoring, factoring by grouping).

ENVIRONMENTS

Virtual Tiles encapsulates three separate environments. The **Addition**, the **Multiplication**, and the **Equations** environment. Only one of these environments is active at any given time. Operations done in any one environment have no effect on the tiles or settings in the other two environments.

When starting the program, the user automatically enters the Addition environment. The menus for each of the environments are described below.

MENUS AVAILABLE IN ALL THREE ENVIRONMENTS

The File Menu

Use this menu for printing the screen or quitting *Virtual Tiles*.

The Edit Menu

Use this menu to clear the screen. The only active item in this menu is the **Erase** item. Selecting this item erases the tiles in the current environment, but does not affect tiles in the other environments. The **Erase** button at the bottom right of the screen serves the same purpose.

The Show Menu

The algebraic representation of placed tiles can be displayed on screen. Use this menu to determine what and how it is displayed. Items in this menu determine whether or not any algebra is displayed, and the nature of the displayed algebra, depending on the current environment.

Algebra Item

When this item is checked, algebra relating to the current environment is displayed, subject to other checked items in this menu (see below). The on-screen **Show Algebra** check box is an alternate means of toggling this menu item.

THE ADDITION ENVIRONMENT

Menus in the Addition Environment

THE SHOW MENU

Addition Item

When this item is checked, the summation of all tiles dragged into the black working area is displayed. For example, if two x tiles and a y tile have been placed in the working area, and this menu item is checked, 2x + y is displayed. The on-screen **Addition** check box is an alternate means of toggling this menu item. (Not available on 12″ monitors).

Simplification Item

When this item is checked, the summation of all tiles dragged into the black working area is simplified and displayed. For example, if three positive x tiles and a negative x tile (double click a placed positive tile to make it negative) have been placed in the working area, and this menu item is checked, 3x - x is simplified to 2x and is displayed. The on-screen **Simplification** check box is an alternate means of toggling this menu item. (Not available on 12″ monitors).

Substitution Item

When this menu is checked, the values for x and y, as set through the pop-up menus immediately below the tile palette, are substituted into the algebra representing the summation of all tiles dragged into the black working area and displayed, but not evaluated. If the simplification item (see previous item) is checked, this substitution is made into the simplified expression. The on-screen **Substitution** check box is an alternate means of toggling this menu item. (Not available on 12″ monitors).

Evaluation Item

When this item is checked, the summation of all tiles dragged into the black working area is evaluated and displayed, using the values for x and y, as set through the pop-up menus immediately below the tile palette. The on-screen **Evaluation** check box is an alternate means of toggling this menu item. (Not available on 12″ monitors).

THE OPTIONS MENU

Group Like Tiles Item

When this item is selected, tiles previously placed in the black working area are sorted and arranged by type. All x tiles are placed together, etc.

Evaluate Tiles Using x Value Item

When this item is selected, tiles previously placed in the black working area that have an x dimension, are replaced with other tiles. The evaluation is done using the value of x as set through the pop-up menu immediately below the tile palette. For example, given that the value of x is set to 5, selecting this menu item would cause an x squared tile to be replaced with 25 unit tiles, an x tile to be replaced with 5 unit tiles, and an xy tile to be replaced with 5 y tiles.

Evaluate Tiles Using y Value Item

When this item is selected, tiles previously placed in the black working area that have a y dimension, are replaced with other tiles. The evaluation is done using the value of y as set through the pop-up menu immediately below the tile palette. For example, given that the value of y is set to 4, selecting this menu would cause a y squared tile to be replaced with 16 unit tiles, a y tile to be replaced with 4 unit tiles, and an xy tile to be replaced with 4 x tiles.

Evaluate Tiles Using x and y Values Item

When this item is selected, tiles previously placed in the black working area that have either an x or a y dimension, are replaced with other tiles. The evaluation is done using the values of x and y as set through the pop-up menus immediately below the tile palette. For example, given that the value of x is set to 2, and the value of y is set to 3, selecting this menu item would cause an x tile to be replaced with 2 unit tiles, a y tile to be replaced by 3 unit tiles, and an xy tile to be replaced with 6 unit tiles.

CHECK BOXES

Show Algebra Check Box

When this box is checked, algebra relating to the current environment is displayed, subject to the state of the other check box items in this environment (see below). When this box is unchecked, the next four check boxes described below are hidden. The **Algebra** item in the Show menu is an alternate means of toggling this check box.

Addition Check Box

When this box is checked, the summation of all tiles dragged into the black working area is displayed. For example, if two x tiles and a y tile have been placed in the working area, and this menu item is checked, 2x + y is displayed. The **Addition** item in the Show menu is an alternate means of toggling this check box. This check box is only visible when the **Show Algebra** check box is checked. (Not available on 12″ monitors).

Simplification Check Box

When this box is checked, the summation of all tiles dragged into the black working area is simplified and displayed. For example, if three positive x tiles and a negative x tile have been placed in the working area, and this menu item is checked, 3x - x is simplified to 2x and is displayed. The **Simplification** item in the Show menu is an alternate means of toggling this check box. This check box is only visible when the **Show Algebra** check box is checked. (Not available on 12″ monitors).

Substitution Check Box

When this box is checked, the values for x and y, as set through the pop-up menus immediately below the tile palette, are substituted into the algebra representing the summation of all tiles dragged into the black working area and displayed, but not evaluated. If the simplification item (see previous item) is checked, this substitution is made into the simplified expression. The **Substitution** item in the Show menu is an alternate means of toggling this check box. This check box is only visible when the **Show Algebra** check box is checked. (Not available on 12″ monitors).

Evaluation Check Box

When this box is checked, the summation of all tiles dragged into the black working area is evaluated and displayed, using the values for x and y, as set through the pop-up menus immediately below the tile palette. The **Evaluation** item in the Show menu is an alternate means of toggling this check box. This check box is only visible when the **Show Algebra** check box is checked. (Not available on 12″ monitors)

POP-UP MENUS

X and Y Pop-up Menus

These pop-up menus are located immediately below the tile palette, and are used to set the values of x and y. These values in turn are used in conjunction with the Show menu (Substitution and Evaluation items) and the Options menu (Evaluate Using x, y, and x and y items). Please refer to the description of these menu items for further details.

THE MULTIPLICATION ENVIRONMENT

Menus in the Multiplication Environment

In addition to the File, Edit and Environment menus, and the Show menu's Algebra item, the following menu items are found under the show menu in the Multiplication Environment.

Width Item

When this item is checked, the sum of all tiles dragged into the black horizontal guide-tile area across the top of the black working area is displayed.

Height Item

When this item is checked, the sum of all tiles dragged into the black vertical guide-tile area along the left side of the black working area is displayed.

Multiplication Item

When this item is checked, the algebra corresponding to the multiplication interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the vertical guide-tile area, a y tile in the horizontal guide-tile area, and an xy tile in the product area, (x)(y) = xy is displayed. The on-screen **Multiplication** radio button is an alternate means of selecting this menu item. (Not available on 12″ monitors).

Factoring Item

When this item is checked, the algebra corresponding to the factoring interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the vertical guide-tile area, a y tile in the horizontal guide-tile area, and an xy tile in the product area, xy = (x)(y) is displayed. The on-screen **Factoring** radio button is an alternate means of selecting this menu item. (Not available on 12″ monitors).

Division by Width Item

When this item is checked, the algebra corresponding to the division by width interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the horizontal vertical guide-tile area, a y tile in the vertical guide-tile area, and an xy tile in the product area, $\frac{xy}{x} = y$ is displayed. The on-screen **Division by Width** radio button is an alternate means of selecting this menu item. (Not available on 12″ monitors).

Division by Height Item

When this item is checked, the algebra corresponding to the division by height interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the vertical guide-tile area, a y tile in the horizontal guide-tile area, and an xy tile in the product area, $\frac{xy}{y} = x$ is displayed. The on-screen **Division by Height** radio button is an alternate means of selecting this menu item. (Not available on 12″ monitors).

THE OPTIONS MENU

Swap Guide Tiles Item

Selecting this item interchanges the tiles in the horizontal guide-tile area with the tiles in the vertical guide-tile area.

Fill in Missing Product Tiles Item

Selecting this item adjusts the tiles in the product area so that they correspond to the product of the tiles in the horizontal and vertical guide-tile areas.

Fill in Missing Guide Tiles Item

Selecting this item adjusts the tiles in the horizontal and vertical guide-tile areas so that when their dimensions are multiplied, the result matches the tiles in the product area. If the product area tiles do not constitute a filled rectangular area, the gaps in the product area are also filled in.

CHECK BOXES

Stay Equal Check Box

When this box is checked within the **Multiplication** environment, placing tiles in any of the black working areas results in the tiles in both the product and the guide areas being adjusted so that the product area corresponds to the product of the tiles in the horizontal and vertical guide-tile areas. **When this check box is checked within the Multiplication environment, tiles already in the product area cannot be moved.**

Show Algebra Check Box

When this box is checked, algebra relating to the current environment is displayed, subject to the state of the other items in this environment (see below). When this box is unchecked, the four radio buttons described below are hidden. The **Algebra** item in the Show menu is an alternate means of setting this check box.

RADIO BUTTONS

Multiplication Radio Button

When this radio button is checked, the algebra corresponding to the multiplication interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the horizontal guide-tile area, a y tile in the vertical guide-tile area, and an xy tile in the product area, (x)(y) = xy is displayed. The **Multiplication** item in the Show menu is an alternate means of setting this radio button. (Not available on 12″ monitors).

Factoring Radio Button

When this radio button is checked, the algebra corresponding to the factoring interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the horizontal guide-tile area, a y tile in the vertical guide-tile area, and an xy tile in the product area, xy = (x)(y) is displayed. The **Factoring** item in the Show menu is an alternate means of setting this radio button. (Not available on 12″ monitors).

Division by Width Radio Button

When this radio button is checked, the algebra corresponding to the division by width interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the horizontal guide-tile area, a y tile in the vertical guide-tile area, and an xy tile in the product area, $\frac{xy}{x} = y$ is displayed. The **Division by Width** item in the Show menu is an alternate means of setting this radio button. (Not available on 12″ monitor).

Division by Height Radio Button

When this radio button is checked, the algebra corresponding to the division by height interpretation of the tiles placed is displayed. For example, if an x tile has been placed in the horizontal guide-tile area, a y tile in the vertical guide-tile area, and an xy tile in the product area, $\frac{xy}{y} = x$ is displayed. The **Division by Height** item in the Show menu is an alternate means of setting this radio button. (Not available on 12" monitor).

THE EQUATIONS ENVIRONMENT

Menus in the Equations Environment

THE OPTIONS MENU

Multiply Both Sides by Item

Selecting this item presents a sub menu of integers ranging from -5 to 5. The tiles in each of the two black working areas (sides of the equation) are multiplied by the number selected.

Divide Both Sides by Item

Selecting this item presents a sub menu of integers ranging from -5 to 5. The tiles in each of the two black working areas (sides of the equation) are divided by the number selected. Divisors which would result in fractional results are inactive.

CHECK BOXES

Stay Equal Check Box

When this box is checked within the **Equations** environment, adding tiles to either of the black working areas (sides of the equation) results in an identical tile being added to the other area. Similarly, if a tile is removed from either side, an identical tile is removed from the other side. If no such tile is available for removal, the additive inverse tile is added to that side. **When this check box is checked within the Equations environment, the sign of a placed tile cannot be changed by double-clicking the tile.**

Show Algebra Check Box

When this box is checked within the equations environment, algebra representing the tiles place on each side of the equation is displayed. The **Algebra** item in the Show menu is an alternate means of toggling this check box.

MANIPULATING TILES

Placing Tiles

Tiles are placed into the black working areas of all environments by dragging tiles from the tile palette in the upper left corner of the screen. Within the **Multiplication** and **Equations** environments, other tiles may also be placed automatically at the same time, should the **Stay Equal** check box be checked. Please refer to the documentation about this check box for further details.

Removing Tiles

Tiles are removed from the black working areas of all environments by dragging placed tiles off of those areas, and dropping them in a non-black region of the screen. Exception #1: If the **Stay Equal** check box is checked within the **Multiplication** environment, tiles cannot be removed from the product area, nor can they be relocated. Exception #2: If the **Stay Equal** check box is checked within the **Equations** environment, if a tile is removed from either side, an identical tile is removed from the other side; if no such tile is available for removal, the additive inverse tile is added to that side.

Changing the Sign of a Placed Tile

Double-clicking on a placed tile flips it's sign. Furthermore, in the **Multiplication** environment, double-clicking tiles in the product area the first time flips the sign of all tiles in that row; a second double-click flips them all back again; a third double-click flips the signs of all tiles in that column; a fourth double-click flips them all back again.

Exception: This feature is disabled in the **Equations** environment when the **Stay Equal** check box is checked.

Canceling Additive Inverses

Placing a tile on top of it's additive inverse results in the two tiles canceling each other out. Exception: This feature is not available in the **Multiplication** environment.

Special Behavior in Multiplication Environment

In the Multiplication Environment, newly placed tiles adopt the sign of previously placed tiles. For example, if a guide tile area already contains a negative x tile, and a positive x tile is added to the same guide area, the new tile will also become negative. Similar behavior occurs when dropping tiles into the product area. Double-clicking tiles in the product area flips the signs of all tiles in that column or row. Please refer to the documentation above about **Changing the Sign of a Placed Tile** for further details.

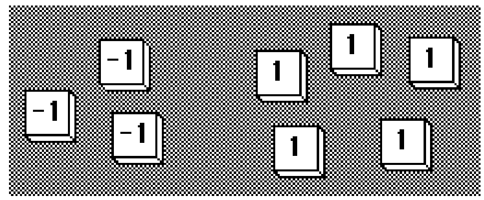
Teacher and Student Activities - The Addition Environment

When you open *Virtual Tiles*, you will automatically be in the 'Addition Environment'. In this environment, students can explore addition of integers, the concept of variable, like–terms, simplification, and evaluation of expressions.

Adding Integers

Example 1

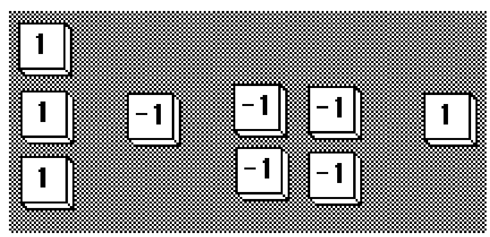
To evaluate –3 + 5, drag three 1 tiles into the working area of the screen (black), and then flip them by double–clicking on them. Now drag in five 1 tiles. This represents –3 + 5.



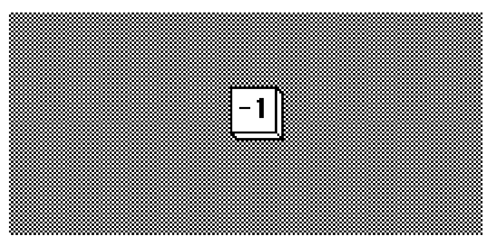
Drag one of the +1 tiles on top of a –1 tile. They should flash for a second and then disappear. This is because –1 + 1 = 0. Continue to do this until there are no more tiles that can be 'eliminated'. This shows that –3 + 5 = 2.

Example 2

Evaluate 3 + (–1) + (–4) + 1
(Drag in the tiles and flip as necessary)



Combine tiles.... and the answer is –1.

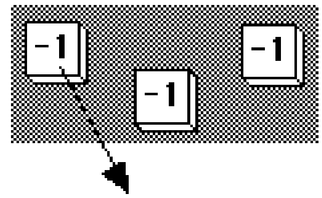


Subtracting Integers

We teach students that subtracting an integer is the same as adding the opposite integer. This idea is easily justified using *Virtual Tiles*.

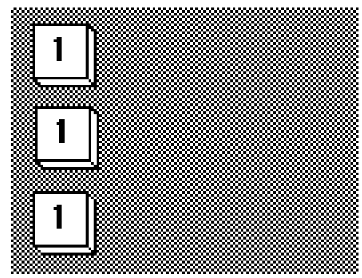
Example 1

Evaluate –3 – (–1)
Drag in –3 into the working area. Interpret this question as -3 'take away' –1. This means we have to remove a –1 tile from the –3. Simple.... drag a –1 tile out of the area, and the answer is –2.



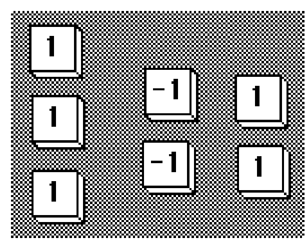
Example 2

Evaluate 3 – 5
This is a more difficult concept. How can you take away 5 when there are only 3 there to begin with? Drag 3 into the working area.

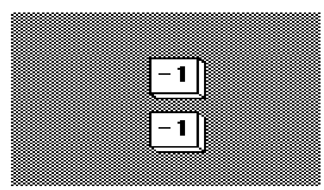


We want to be able to take away 5, but can't... yet! We could add 0 to the question without changing it, so...

Add two +1 tiles and two –1 tiles to the area. (To get a –1 tile, drag in a 1 tile and then double–click on it to flip it over.) The reason we added two of each was so that we would now have enough +1 tiles in order to take away 5 of them.



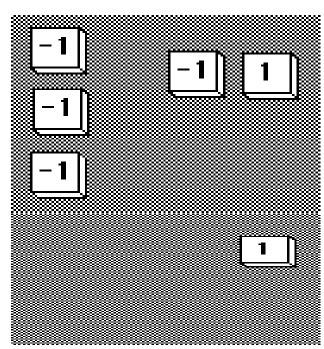
When we take away 5 (i.e., drag five +1 tiles out of the area), the answer is –2.



Example 3

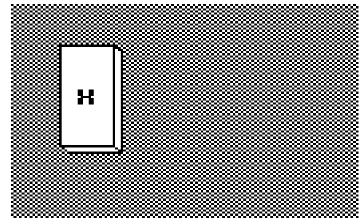
Evaluate –3 – (–4)
Add in one –1 tile and one +1 tile in order to have enough –1 tiles to take them away.

Now 'take–away' –4 from the area, and the answer is 1.

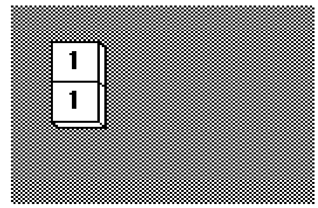


Variables and Substitution

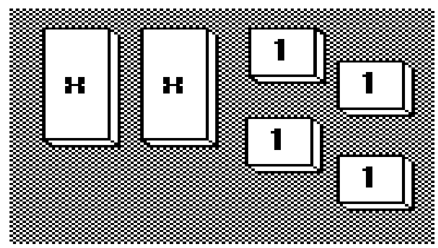
Example 1



Drag an x–tile into the working area (black). On the left of the screen you will see two pop-up menus, x = [0]   y = [0] . This means that the value of x and y are automatically set to 0 to begin with. Both x and y are variables, so they can be set to any number you like. Change the value of x to 2 by clicking (and holding down the mouse) on the box with 0 in it, and then moving to the number you'd like to change it to. Under the 'Options' menu at the top of the screen, select 'Evaluate tiles using x value'. Instantly the x tile is replaced with 2 unit tiles.

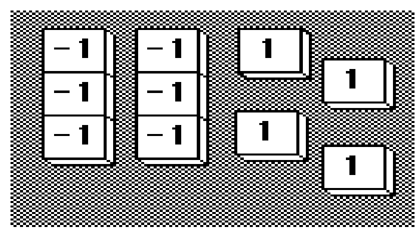


Example 2

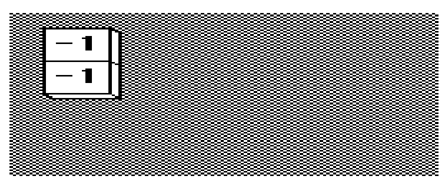


Evaluate $2x + 4$ if $x = -3$
Drag in two x tiles and four 1 tiles. This represents $2x + 4$.

Set the value of x to –3 using the pop-up menus. Under 'Options', select 'Evaluate using x value'. The working area will appear as it is on the right.

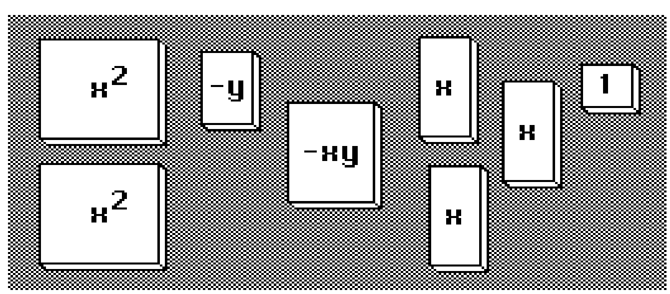


Drag the –1 tiles on top of the +1 tiles to eliminate them. The answer is –2.

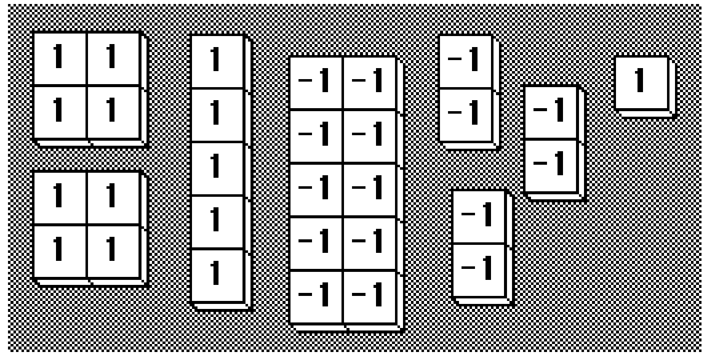


Example 3

Evaluate $2x^2 - y - xy + 3x + 1$
if $x = -2$ and $y = -5$
Drag the tiles over as shown.



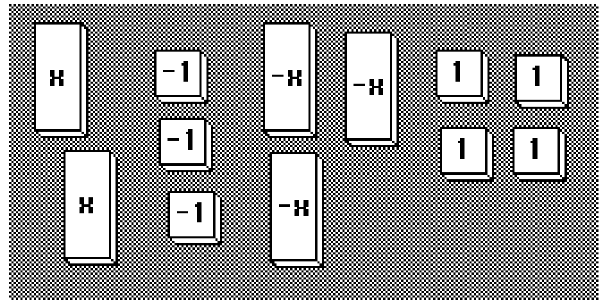
Now, change the value of x to –2 and y to –5, and then select 'Evaluate using x & y tiles' under 'Options'. Notice that x^2 has been replaced by +4, –y by +5, –xy by –10, and each x by –2.
Eliminate 1's and –1's.
The answer is –2.



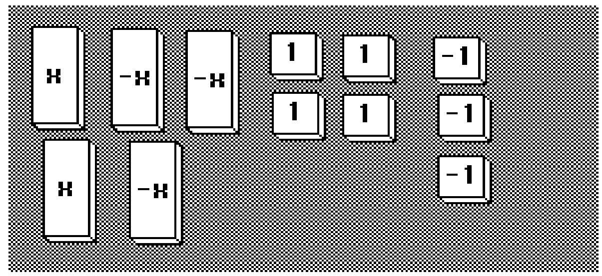
Combining Like Terms

Example 1

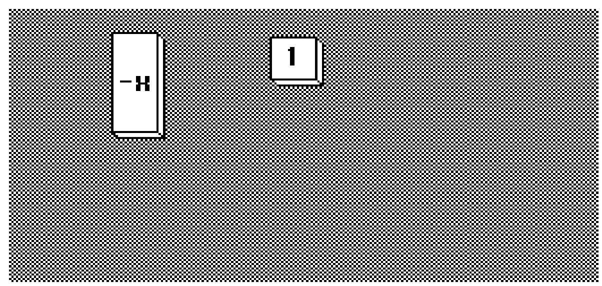
Simplify $2x - 3 - 3x + 4$.



- Drag the tiles into the working area as shown. Remember that to get a negative tile, drag in a positive tile and then double–click on it to flip it over.



- Rearrange the tiles so that the 'like' tiles (tiles that are the same size) are together.



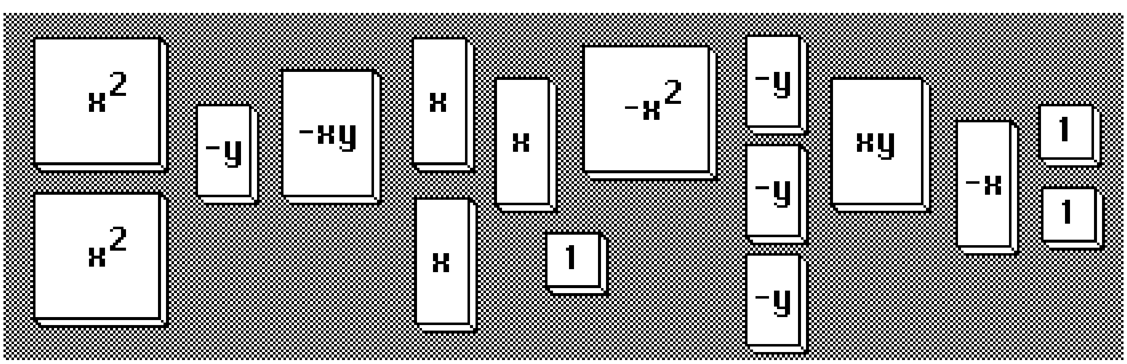
- Drag one of the $-x$ tiles on top of an x tile. It should flash for an instant and then disappear since $-x + x = 0$. Repeat for any other tiles which will eliminate each other. We find that...

$$2x - 3 - 3x + 4 = -x + 1$$

Example 2

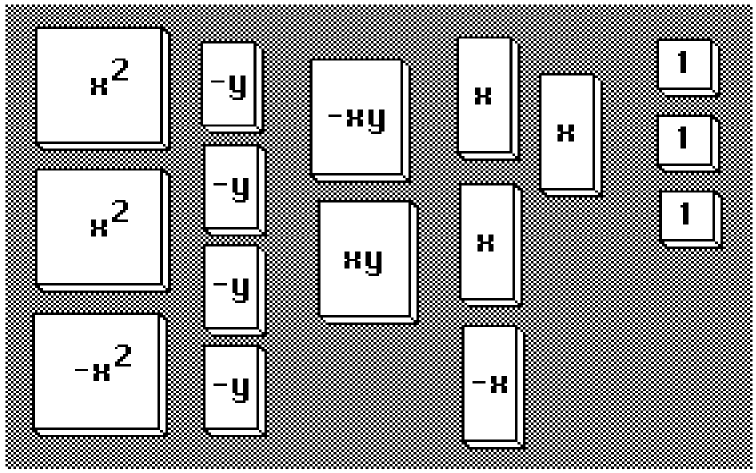
Simplify $2x^2 - y - xy + 3x + 1 - x^2 - 3y + xy - x + 2$

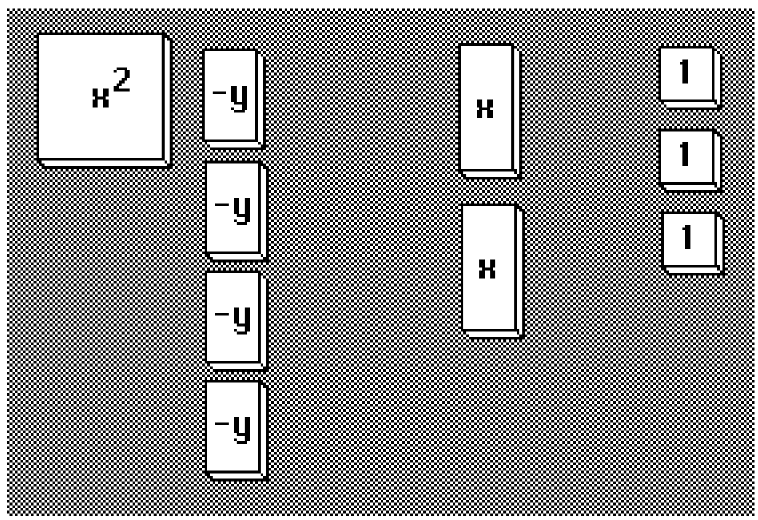
Drag the tiles into the working area as shown below.



Variables and Substitution

Rearrange the tiles so that the 'like' tiles are together, or select 'Group Like Tiles' under 'Options'.





Drag the $-x^2$ tile on top of the x^2 tile. It should flash for an instant and then disappear since $-x^2 + x^2 = 0$. Similarly, combine $-xy$ with xy and $-x$ with x. The result cannot be simplified any more. So the final answer is

$$x^2 - 4y + 2x + 3$$

Student Practice Sheet

Name:______________________________
Date: ______________________________

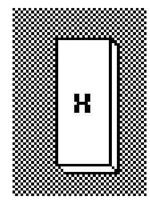
Evaluation of an Expression

In math, we use variables (letters) to represent unknown numbers. For example, a taxi might charge you $1 as soon as you get in and then $2 per kilometre after that. The total amount of money owed to the driver will be $1 + $2 times the number of kilometres driven. If the number of kilometres is unknown, then we could use x to represent it. The amount owed (in $) will be $1 + 2x$.

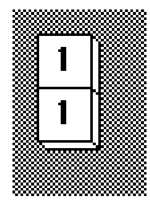
You will now see how this expression can be formed using *Virtual Tiles*.

Using instructions from your teacher, turn on the computer, and open the *Virtual Tiles* program (by double clicking on the icon). Notice the pictures of rectangular blocks in the top left corner. These are the tiles you will use to do much of your algebra.

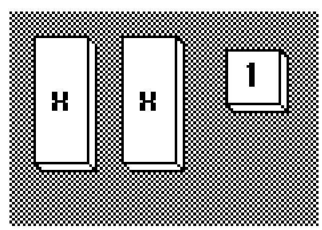
Example 1



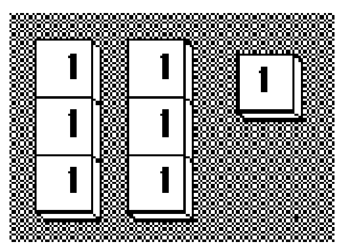
Drag an x–tile into the working area (black). On the left of the screen you will see two pop-up menus x = [0]   y = [0] . This means that the value of x and y are automatically set to 0 to begin with. Both x and y are variables, so they can be set to any number you like. Set the value of x to 2 by clicking (and holding down the mouse) on the box with 0 in it, and then moving to the number you'd like to change it to.
Under the 'Options' menu at the top of the screen, select 'Evaluate Tiles using x Value'. Instantly the x–tile is replaced with 2 of the 1–tiles.



Example 2

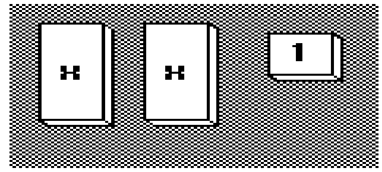


Evaluate $2x + 1$ if $x = 3$. (This means we want to work out the total value of $2x + 1$ if each x is replaced with 3.) Drag in two x–tiles and a 1–tiles. This 'is' $2x + 1$.

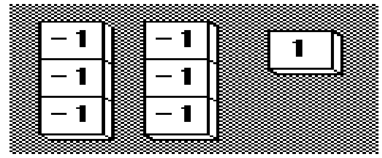


Change the value of x (by clicking on the x= box) to 3, and then select 'Evaluate Tiles using x Value'. The working area will appear as it is on the right. Each of the x–tiles has been replaced with 3 of the 1–tiles. The total 'value' is 7.

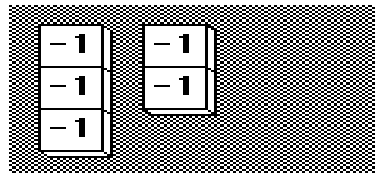
Example 3
Evaluate $2x + 1$ if $x = -3$
Erase. Drag in two x–tiles and a 1–tile.



Under 'Options', select 'Evaluate Tiles using x Value', and then change the value of x (by clicking on the x= box) to –3. The working area will appear as it is on the right.



Drag the 1 tile on top of one of the –1 tiles. They will flash for a second and then disappear because $-1 + 1 = 0$. The final answer is –5.



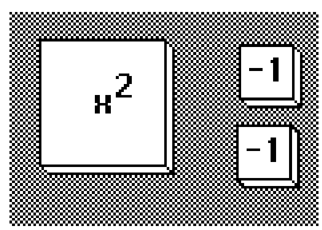
So far we have found that $2x + 1$ has a value of 7 if $x = 3$ and a value of –5 if $x = -3$. We can put these answers into a **table of values** as shown on the right.

x	$2x+1$
–3	–5
–2	
–1	
0	
1	
2	
3	7

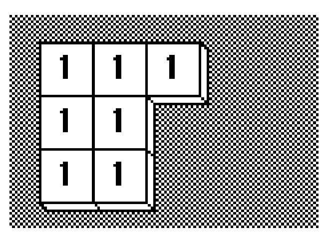
Repeat the steps above to evaluate $2x + 1$ when x is each of –2, –1, 0, 1, and 2. Record your answers in the table.

$2x + 1$ is called a variable expression because its value can change depending on the value of the variable.

Example 4 Evaluate $x^2 - 2$ if $x = 3$.
Erase. Drag in an x^2–tile, and two 1–tiles. Double click on each of the 1–tiles to make them negative. This builds the expression $x^2 - 2$. Note that an x^2–tile is square with each side the same length as an x–tile. We know that x^2 means x times x.



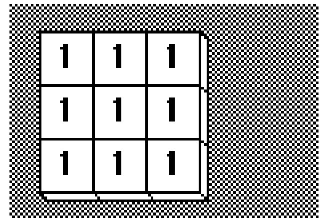
If x is 3, then x^2 is 3 x 3. This can be shown using a square which is 3 rows of 3 (a 3 by 3 square...) Change the value of x to 3 and choose 'Evaluate using x value'. Eliminate the two –1 tiles by dragging them on top of 1–tiles. The result is 7 (as shown).



Example 5

Evaluate x^2 if $x = -3$.

Repeat as above. If x is –3, then x^2 is –3 x –3. We know a negative number multiplied by a negative number will be positive, so the answer will be 9.



x	$3x - 2$
–3	
–2	
–1	
0	
1	
2	
3	

Questions

1. a) Complete the following table of values for the expression $3x - 2$ using Virtual Tiles.

   b) Can you find a pattern in the numbers on the right side of your table? Describe it.

x	$x^2 - 2$
–3	
–2	
–1	
0	
1	
2	
3	

2. a) Complete the following table of values for the expression $x^2 - 2$ using Virtual Tiles.

   b) Can you find a pattern in the numbers on the right side of your table? Describe it.

3. Evaluate $2x^2 - y - xy + 3x + 1$ if $x = -2$ and $y = -5$ using Virtual Tiles.

4. a) Complete the table shown below by evaluating the expression $3x - 2y$ for each of the combinations of x and y listed. E.g., the box in the top right corner of the table will get the answer from evaluating $3x - 2y$ if $x = 3$ and $y = -3$ (the answer is 15). Some of them have been done for you already!

		Values for x						
Table for $3x - 2y$		$x = -3$	$x = -2$	$x = -1$	$x = 0$	$x = 1$	$x = 2$	$x = 3$
Values for y	y = –3							15
	y = –2	–5		1		7		
	y = –1	–7						11
	y = 0	–9	–6	–3	0	3	6	9
	y = 1	–11						7
	y = 2	–13		–7		–1		
	y = 3	–15	–12	–9	–6	–3	0	3

b) Describe as many patterns as you can find in the table.

5. a) Make a graph using each pair of numbers in the table in question 1 as an ordered pair for a point. i.e., use the answers from $3x - 2$ as the y value for the point. The first point on the graph will be at (–3, –11). Draw a line through the points. This is the graph of the equation $y = 3x - 2$.

   b) Determine the slope of the line (use any two points). Compare that to the expression $3x - 2$ and the patterns you noticed above.

   c) Where does the line cross the y–axis? How might you get that answer from the equation $y = 3x - 2$?

   d) Use a graphing program to graph the line with equation $y = 3x - 2$.

   e) Open a spreadsheet. Create the same table of values above using the spreadsheet. Have the computer work out the answers rather than typing them.

6. a) Make a graph using each pair of numbers in the table in question 2 as an ordered pair for a point. i.e., use the answers from $x^2 - 2$ as the y value for the point. The first point on the graph will be at (–3, 7). Draw a smooth curve through all of the points.

   b) Compare the patterns you noticed before with the graph. Why would you expect it to look the way it does?

   c) From the graph, what is the lowest number that y can ever be? How might you get that answer from the equation $y = x^2 - 2$?

   d) Use a graphing program to graph the curve with equation $y = x^2 - 2$.

   e) Open a spreadsheet. Create the same table of values above using the spreadsheet. Have the computer work out the answers rather than typing them.

7. a) Circle each cell in the table of values you created in question 4 which worked out to be equal to 0. Make a graph using the value of x and of y from that cell for each point on the graph. Draw a straight line through the points. This is the line with equation $3x - 2y = 0$.

   b) Use a graphing program to graph this line.

Student Practice Sheet

Name:________________________
Date: ________________________

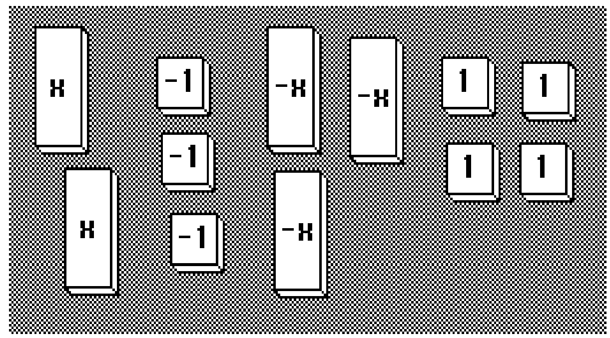
Combining Like Terms

Using instructions from your teacher, turn on the computer, and open the *Virtual Tiles* program (by double clicking on the icon). Note the pictures of rectangular blocks in the top left corner. These are the tiles you will use to do much of your algebra.
Today you will discover how to simplify algebraic expressions by combining **like terms**.

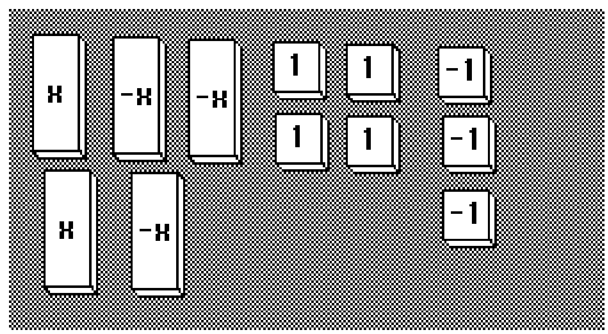
Example 1

Simplify $2x - 3 - 3x + 4$.

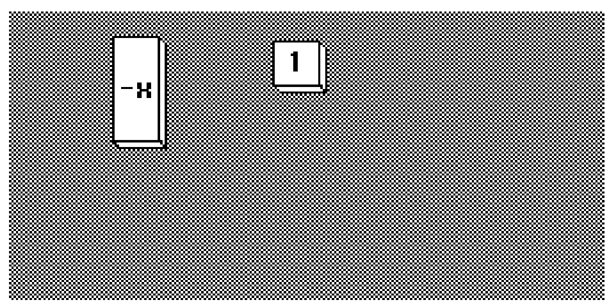
- Drag the tiles into the working area as shown. Remember that to get a negative tile, drag in a positive tile and then double–click on it to flip it over.



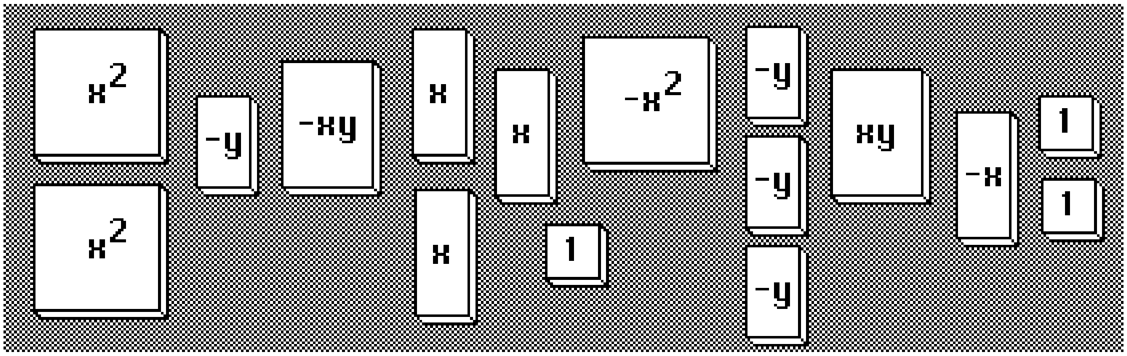
- Rearrange the tiles so that the 'like' tiles (tiles that are the same size) are together.



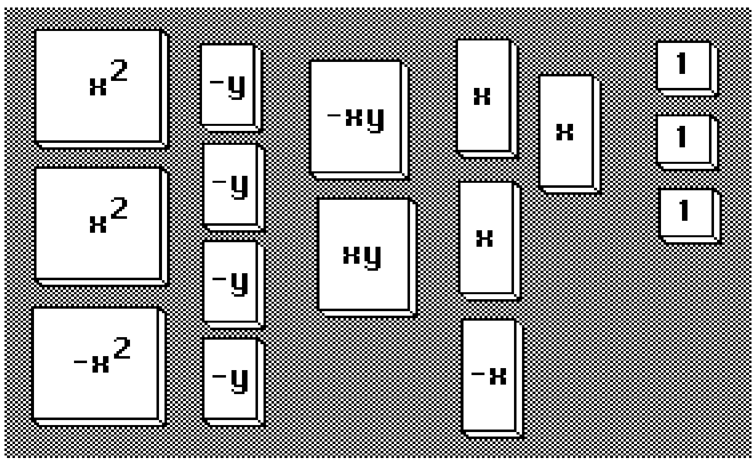
- Drag one of the $-x$ tiles on top of an x tile. It should flash for an instant and then disappear since $-x + x = 0$. Repeat for any other tiles which will eliminate each other. When you can no longer eliminate any terms in this way you are done. You should find that... $2x - 3 - 3x + 4 = -x + 1$ as shown on the right.



Example 2 Simplify $2x^2 - y - xy + 3x + 1 - x^2 - 3y + xy - x + 2$
Drag the tiles into the working area as shown below.

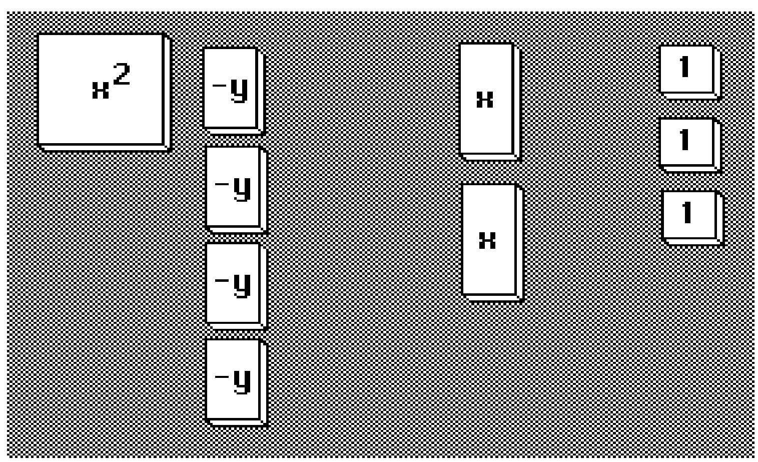


Rearrange the tiles so that the 'like' tiles are together, or select 'Group Like Tiles' under 'Options'.



Drag the $-x^2$ tile on top of the x^2 tile. It should flash for an instant and then disappear since $-x^2 + x^2 = 0$. Similarly, combine $-xy$ with xy and $-x$ with x. The result cannot be simplified any more. So the final answer is

$x^2 - 4y + 2x + 3$



Questions

1. a) Simplify $-x + 3 + 4x - 3$.
   b) Evaluate $-x + 3 + 4x - 3$ when $x = -2$.
   c) Evaluate $3x$ when $x = -2$.
   d) Use *Virtual Tiles* to help you complete the following table of values.

x	$-x + 3 + 4x - 3$	$3x$
-3		
-2		
-1		
0		
1		
2		
3		

   e) Why do you think the numbers in the table worked out the way they did?

2. a) Simplify $-x + 3 - 2x - 5 + 3x + 3$
   b) **Terms** in an expression are called **like terms** if you can combine them by adding or subtracting. How do you know if *Virtual Tiles* can be added or subtracted?
   c) Which of the following terms are like terms with $-2x$? (Circle them)

   $3x$ $-y$ x^2 $5xy$ $3y$ $-x$ xy -2 5 $2x$ $3xy$

3. a) Simplify $-x^2 + 2x + 5 + 3x^2 - 3x - 4$
   b) Which of the following terms are like terms with $-2x^2$? (Circle them)

   $3x$ $-y$ x^2 $5xy$ $3y$ $-x$ $3x^2$ -2 5 $2x$ $3xy$

4. a) Simplify $xy - x - 3xy - y^2 + x + 2x^2 - x^2 + y^2 - 3x - 4$
   b) Which of the following terms are like terms with $-2xy$? (Circle them)

   $3x$ $-y$ x^2 $5xy$ $3y$ $-x$ xy -2 5 $2x$ $3xy$

5. a) Make up an expression and write it down.
   b) Simplify it using *Virtual Tiles*.
   c) How could you do this without *Virtual Tiles*?

6. a) Make up an expression which would simplify to become $3x - 1$ and write it down.
   b) Make up another expression which would simplify to become $3x - 1$ and write it down.
   c) Make up another expression which would simplify to become $3x - 1$ and write it down.
   d) What answer would you get if you evaluated each of your expressions when $x = 5$? Why?

Teacher and Student Activities - The Equation Environment

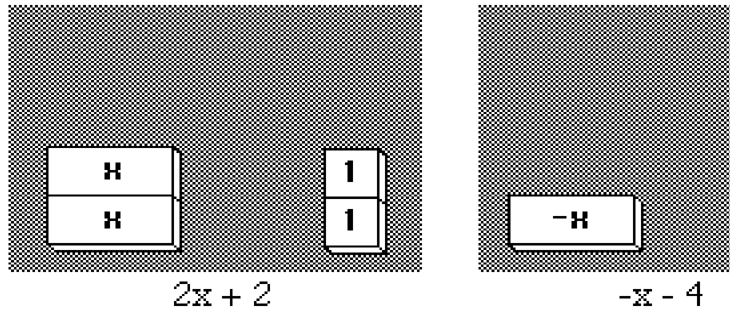
Under the 'Environment' menu, select 'Equations'.

Solving Linear Equations

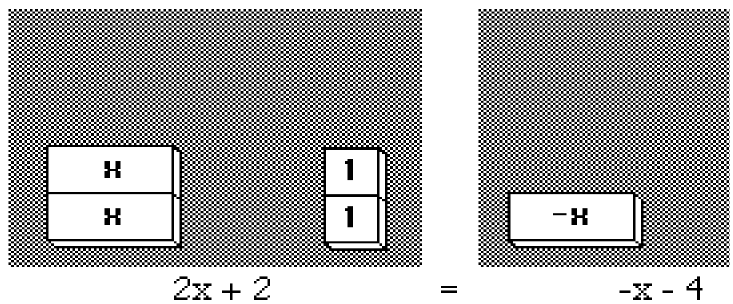
Example 1

Solve $2x + 2 = -x - 4$

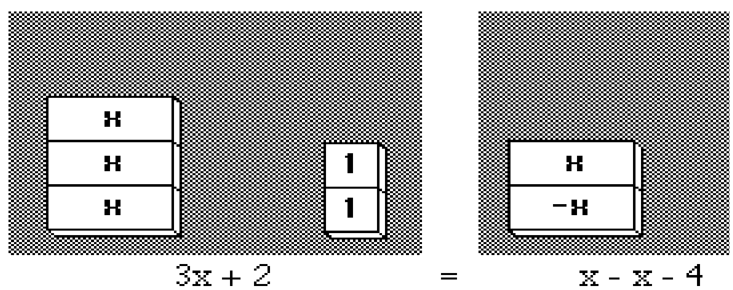
- Drag the tiles into the two different sides of the equation balance as shown. Notice the expressions underneath each side. This will always appear unless you turn off the 'Show Algebra' check box.



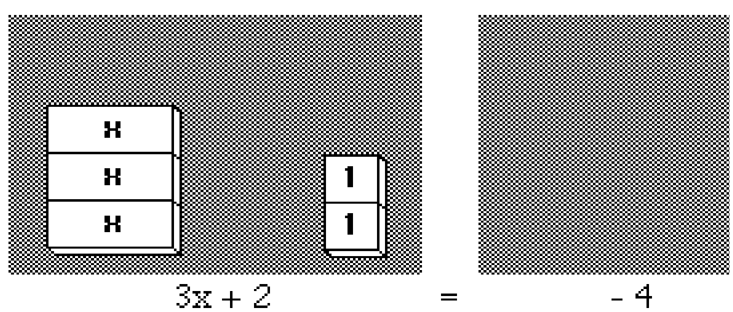
- Now, click on the 'Stay Equal' check box. Notice that an equal sign now appears between the expressions on the two sides. From now on, anything you add to one side of the equation will be added to both sides.



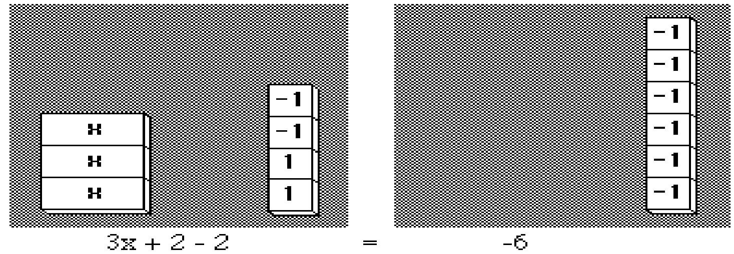
- Eliminate the $-x$ on the right side by adding an x–tile to each side by dragging an x–tile into the left side. As you do that, an x will also be added on the right side. The equation has been automatically updated.



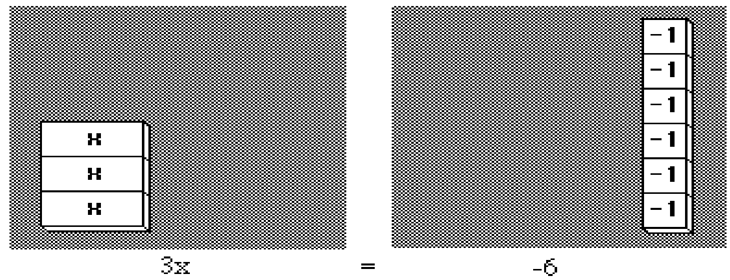
- Drag the x tile on top of the $-x$ tile on the right side. They should flash for an instant and then disappear since $-x + x = 0$.



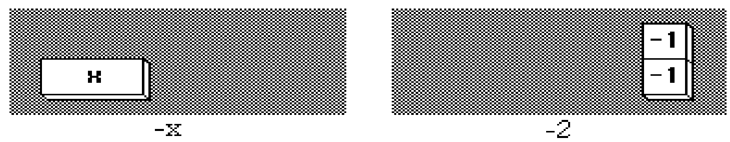
- Eliminate the two 1 tiles on the left side by adding two –1 tiles to each side. Drag a couple of –1 tiles into the left side. As you do that, –2 will be added on the right side.



- Drag the –1 tiles on top of the 1–tiles on the left to eliminate them.



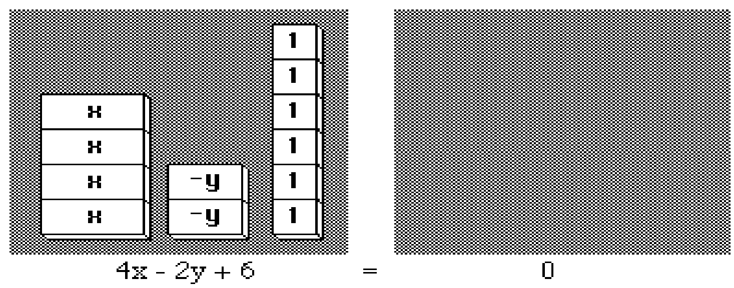
- Under 'Options', select 'Divide both sides by ...', hold the mouse down and drag down to 3 to divide both sides by 3. The result is as shown.
  i.e., the answer is $x = -2$.



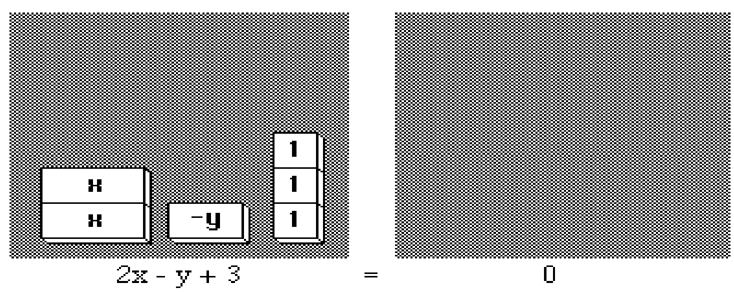
Example 2

Solve $4x - 2y + 6 = 0$ for y.

- Drag the appropriate tiles over as shown and click on the 'Stay Equal' button.



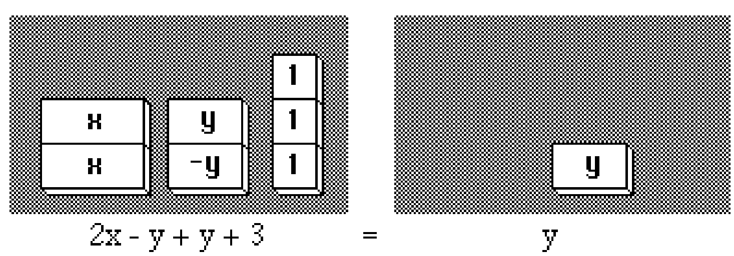
- At this point you have several choices, however the easiest thing to do is to notice that each side can be divided by 2 at this point. Under 'Options' select divide both sides by 2.



- Add y to both sides to eliminate –y.

- Drag the y tile on top of the –y tile on the left side.

The result is that $y = 2x + 3$



Student Practice Sheet

Name:______________________________
Date: ______________________________

Solving Equations

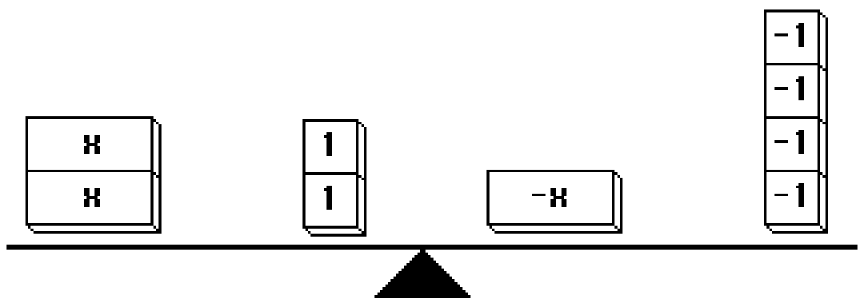
Using instructions from your teacher, turn on the computer, and open the *Virtual Tiles* program (by double clicking on the icon).

Today you will discover how to solve equations using Virtual Tiles. Both of 2x + 2 and –x – 4 are variable expressions. If x is equal to different numbers, 2x + 2 and –x – 4 will also change value. To solve the equation 2x + 2 = –x – 4 means to find the number that x must be in order that the two expressions are equal to each other.

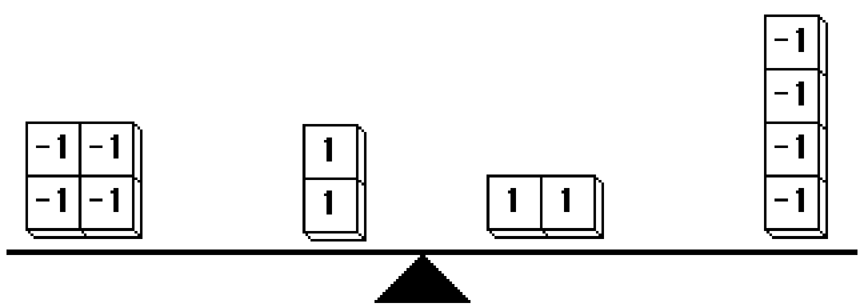
For example, if x = 3, then the value of 2x + 2 is 8 and the value of –x – 4 is –7. … x=3 is NOT the correct answer to the equation.

Can you guess what the answer is?

We sometimes think of an equation as a balance or a teeter–totter. We are trying to find a number to replace x with in order to keep the teeter–totter balanced.



If x is replaced by –2, the balance would look like this...



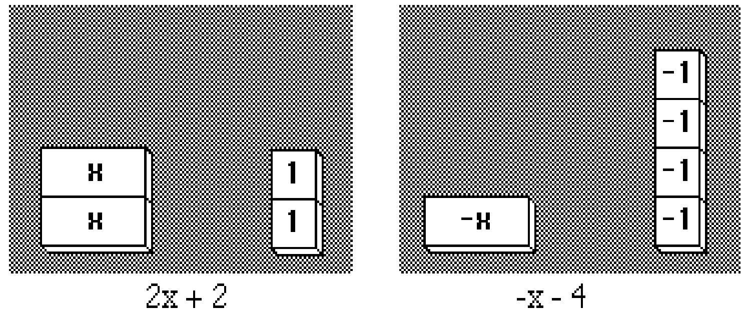
The left side works out to be equal to the right side, therefore the **root** (answer) of the equation is x = –2

We can use *Virtual Tiles* to solve the equation. Because it is a balance, we can add or subtract the same quantities from both sides of the equation. We can also multiply or divide both sides of the equation by the same amount.

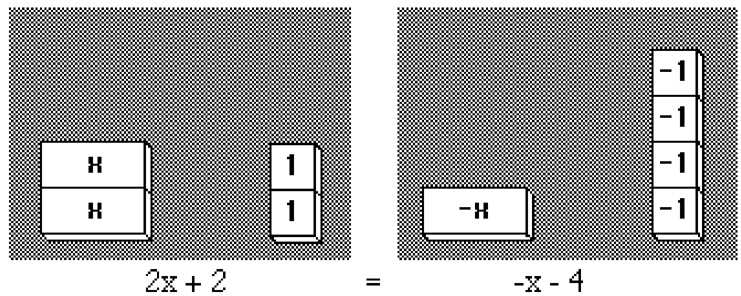
If you are in *Virtual Tiles*, select 'Equations' from the 'Environment' menu.

Example 1

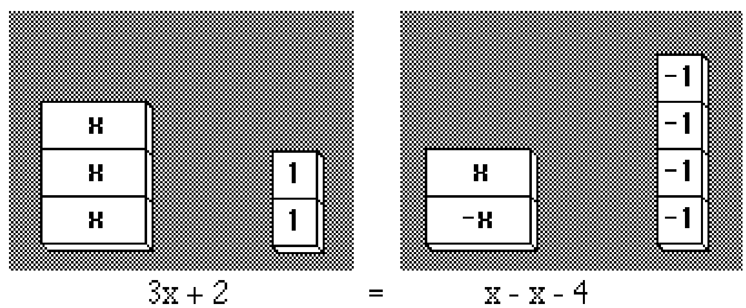
To solve $2x + 2 = -x - 4$



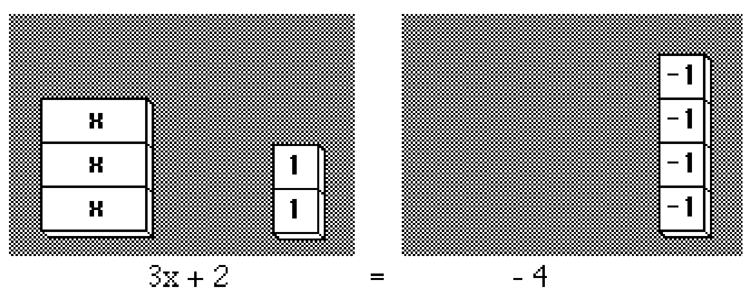
- Drag the tiles into the two different sides of the equation balance as shown. Notice the expressions underneath each side. This will always appear unless you turn off the 'Show Algebra' check box.



- Now, click on the 'Stay Equal' check box. Notice that an equal sign now appears between the expressions on the two sides. From now on, anything you add to one side of the equation will be added to both sides.

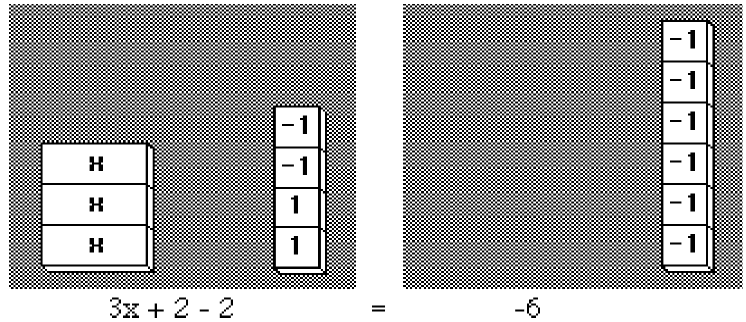


- Drag an x tile into the right side. As you do that, an x will be added on the right side. We are doing that because the right side has a –x tile that we would like to get rid of. Notice that the equation underneath is automatically updated.

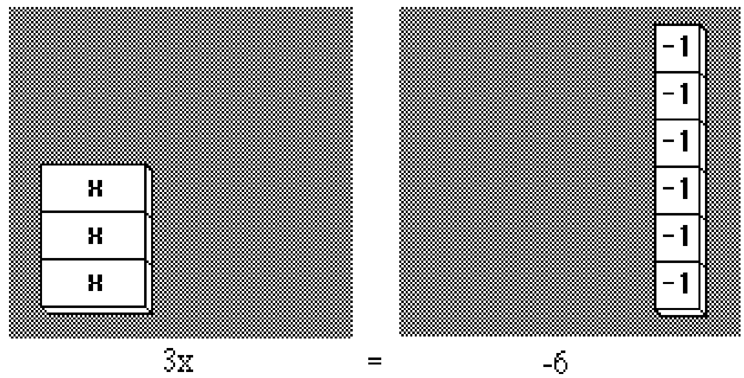


- Drag the x tile on top of the $-x$ tile on the right side. They should flash for an instant and then disappear since $-x + x = 0$. The equation is updated again.

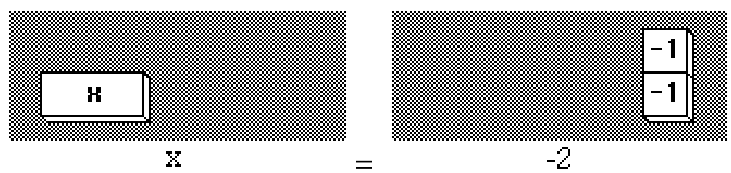
- Eliminate the two 1 tiles on the left side by adding two –1 tiles to each side. Drag a couple of –1 tiles into the left side. As you do that, –2 will be added on the right side.



- Drag the –1 tiles on top of the 1–tiles on the left to eliminate them.



- Under 'Options', select 'Divide both sides by ...', hold the mouse down and drag down and select 3 to divide both sides by 3. The result is as shown. I.e., the answer is $x = -2$.



Questions

1. a) Solve $-x + 3 = -2x - 1$
   b) Evaluate $-x + 3$ using the answer you got for x in part a).
   c) Evaluate $-2x - 1$ using the answer you got for x in part a).
   d) Explain your result.

2. a) Solve $1 + 4x = 2x + 1$
   b) Evaluate the left side of the equation using the answer you got for x in part a).
   c) Evaluate the right side of the equation using the answer you got for x in part a).
   d) Explain your result.

3. a) Solve $1 - x - 3 = 2x + 4$
   b) Evaluate the left side of the equation using the answer you got for x in part a).
   c) Evaluate the right side of the equation using the answer you got for x in part a).
   d) Explain your result.

4. a) Solve $5 - 2x = -3$
   b) Evaluate the left side of the equation using the answer you got for x in part a).
   c) Evaluate the right side of the equation using the answer you got for x in part a).
   d) Explain your result.

5. a) Solve $3y - 3 = 2y$
   b) Evaluate the left side of the equation using the answer you got for y in part a).
   c) Evaluate the right side of the equation using the answer you got for y in part a).
   d) Explain your result.

6. a) Solve $2y - x - 3 = -x + 5$
   b) Why does your answer not tell you what x is equal to?

7. a) Solve $3 + 2x = 3x + 4 - x$
   b) Explain your result.

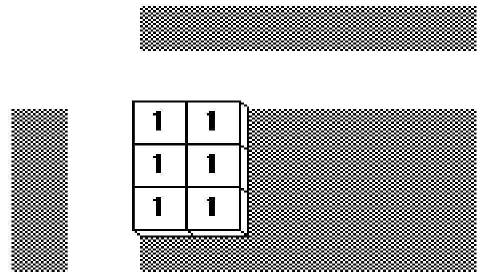
8. a) Solve $3 + 2x = 3x + 3 - x$
   b) Explain your result.

9. a) Try to solve $3 + 2x = 2$ using Virtual Tiles.
   b) Although *Virtual Tiles* will not let you finish solving this equation, can you state the answer to the equation anyway?

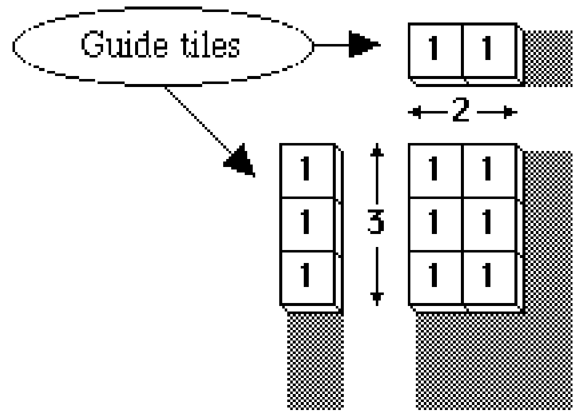
10. Make up your own equation and solve it using *Virtual Tiles*. Be as creative as possible.

Multiplication of Integers

- We know that 3 **X** 2 means 3 groups of 2, so it makes sense to show 3 **X** 2 as follows:



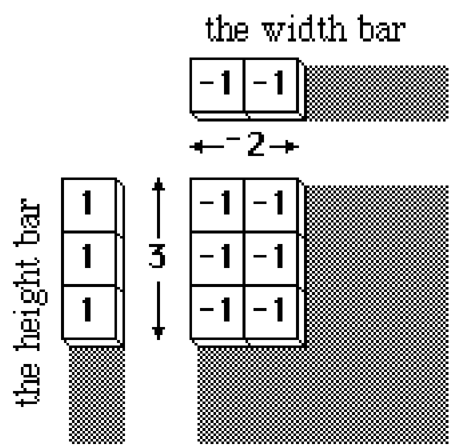
- We can add 'guide tiles' in the width and height bars to show how long each side of the rectangle is

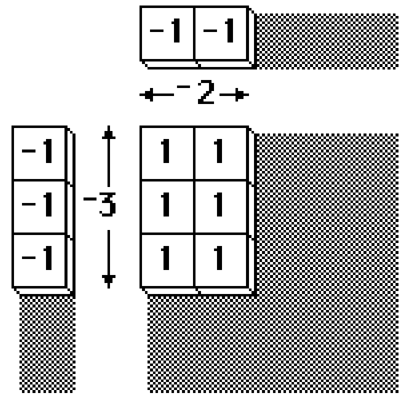


- To multiply using *Virtual Tiles*, open the 'Environment' menu at the top of the screen by clicking and holding down the mouse. Drag the mouse down and select 'Multiplication'.
- Click on the 'Stay Equal' check box on the left.
- Place the tiles you want to multiply in the guide bars as shown above by dragging them over.
- If you want the computer to display the answer, make sure the 'Show Algebra' box has an X in it, and the radio button beside 'Multiplication' is black (or that 'multiplication' is checked in the Show menu).

- Now, double–click on one of the tiles in the width bar. Both tiles in the top row and all of the tiles below it should flip as shown on the right.

  This shows us that –2 **X** 3 = –6.



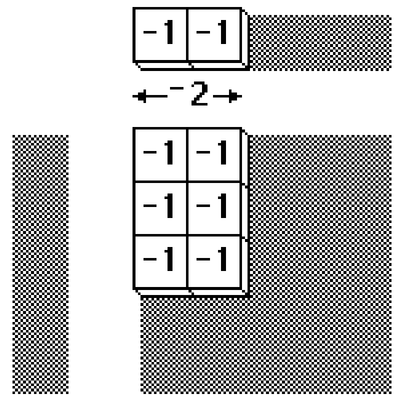


- Now double–click on one of the tiles in the height bar.
  This shows that –2 **X** (–3) = 6. (The tiles in the product area are flipped twice!)

Division of Integers

To divide a number by another one, place the number of tiles you want to divide by (the divisor) in the width or height bar, and then place the number you want to divide (the dividend) into the product area. Rearrange the tiles into a rectangle with one side matching the guide tiles. Now fill in the missing guide yourself or click on the 'Stay Equal' check box.

E.g., to divide –6 by –2, arrange the tiles as shown, and then fill in the tiles in the height bar or click on 'Stay Equal'.

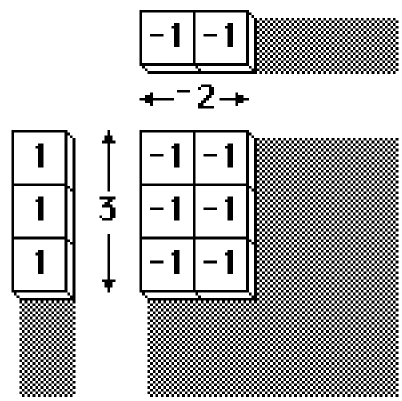


The result is shown on the right. We would then conclude that

$$-6 \div (-2) = 3$$

If you click on the 'division by width' radio button, (or select this item from the show menu) it shows that

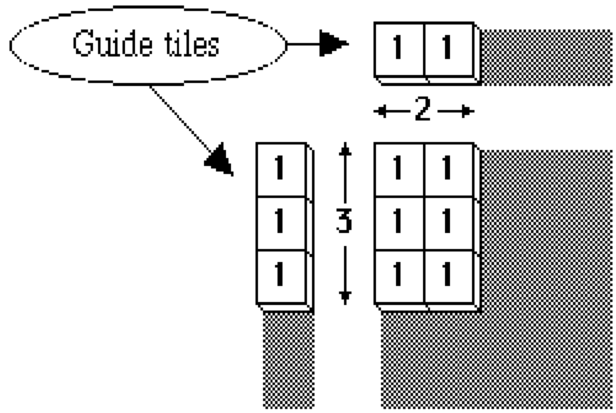
$$\frac{-6}{-2} = 3$$



Using this model, it is easy for students to understand why a negative number divided by another negative number must be positive.

Multiplication of Polynomials

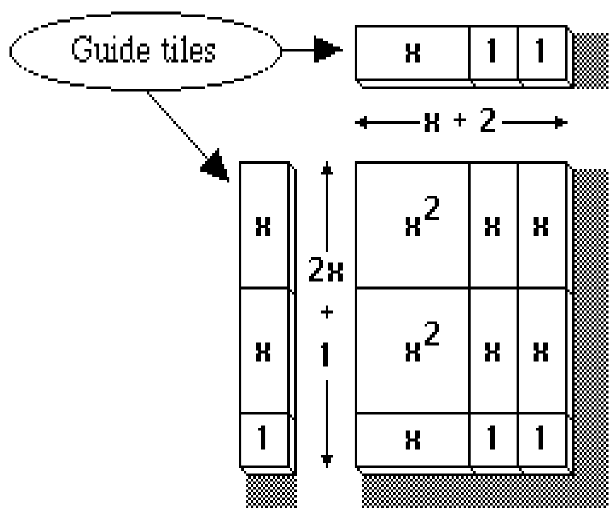
We know that 3 **X** 2 means 3 groups of 2, so it makes sense to show 3 **X** 2 as shown on the right.



The 'guide tiles' in the width and height bars show how long each side of the rectangle is.

To do this using *Virtual Tiles*, make sure there is an X in the 'Stay Equal' box, and then place the tiles you want to multiply in the guide bars as shown on the right.

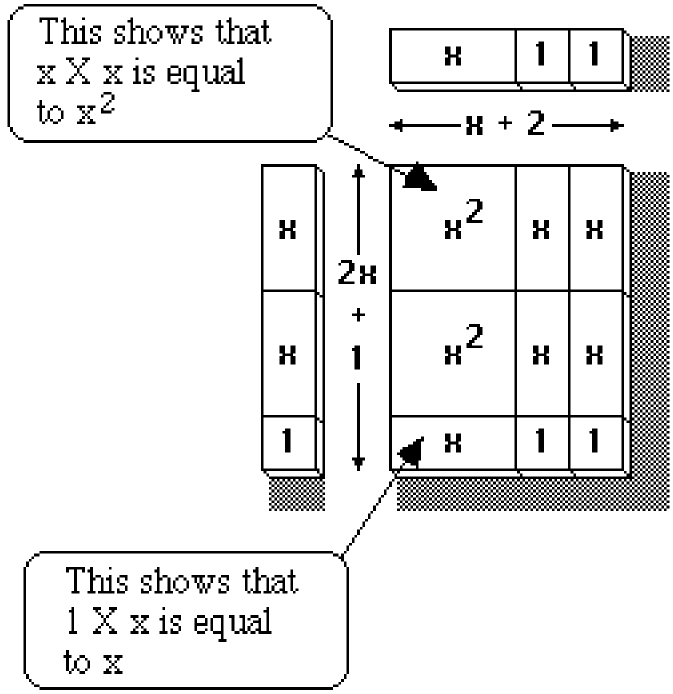
If you want the computer to display the answer, make sure the 'Show Algebra' box has an X in it, and the button beside 'Multiplication' is black.

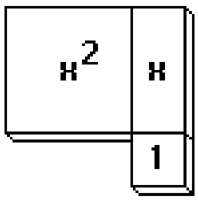


- For example, to multiply $x - 2$ by $2x + 1$, make sure there is an X in the 'Stay Equal' box, fill the guide bars with the x and 1 tiles as shown on the right. (Ignore the negative sign for now.)

Notice that:

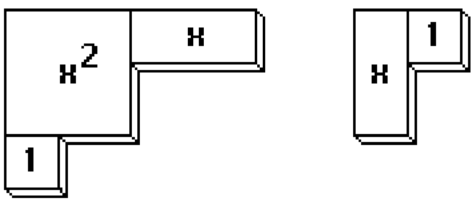
- the edges of the guide tiles line up perfectly with the edges of the product tiles.
- each product tile is the answer from the multiplication of two guide tiles (see example shown)
- if two tiles touch each other along a side, the sides must be the same length.



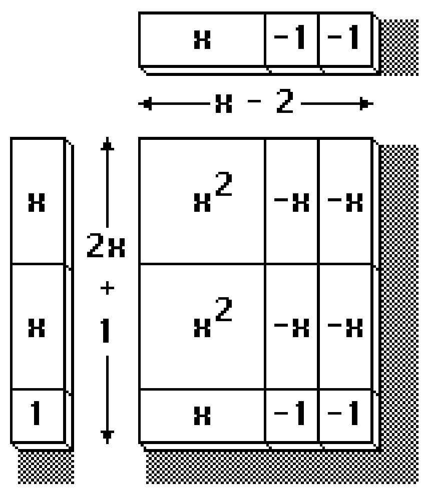


This is okay because the shared sides are identical, but ...

neither of these is possible



- Now, double–click on one of the 1–tiles in the width bar. Both 1–tiles in the top row and all of the tiles below it should flip as shown on the right.

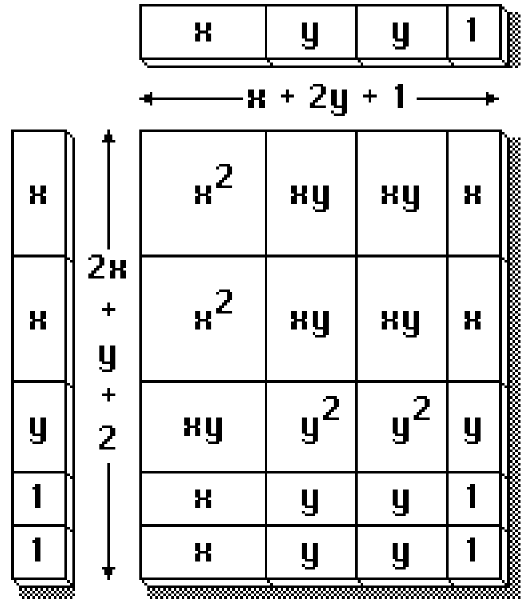


This shows us that $(2x + 1)(x - 2)$

$= 2x^2 + x - 4x - 2$

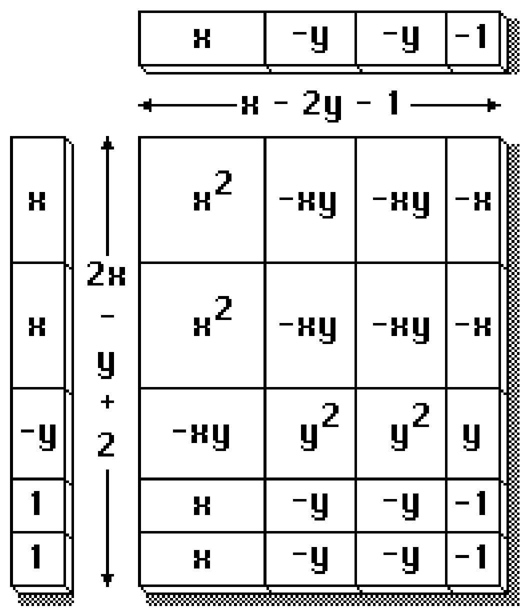
$= 2x^2 - 3x - 2$

If you want the computer to display the answer, make sure the 'Show Algebra' check box has an X in it, and the radio button beside 'Multiplication' is black, (or that this item is checked in the Show menu). If you had chosen this at the beginning, it would have updated the equation each time you dropped a tile into any area.



To multiply (x – 2y – 1) by (2x – y + 2),

- drop the appropriate tiles into the width and height bars,
- fill in the rectangle formed by the guide tiles with positive tiles (note that this requires y, xy, and y^2 terms as shown) or click on the 'Stay Equal' check box.



- Now ... flip the guide tiles that should be negative.

- Decide on the sign of each product tile by thinking about the number of times each has been flipped. (Or ... the computer will do it if you clicked on 'Stay Equal')

- Collect like terms... the answer is
  $2x^2 - 5xy - 2x + 2x + 2y^2 + y - 4y - 2$
  $= 2x^2 - 5xy + 2y^2 - 3y - 2$

  If the 'Show Algebra' box has an X in it, and the button beside 'Multiplication' is black, then this expansion will be displayed.

This example reinforces the concepts of combining like terms. Tiles which are not alike cannot be combined (they are not the same size).

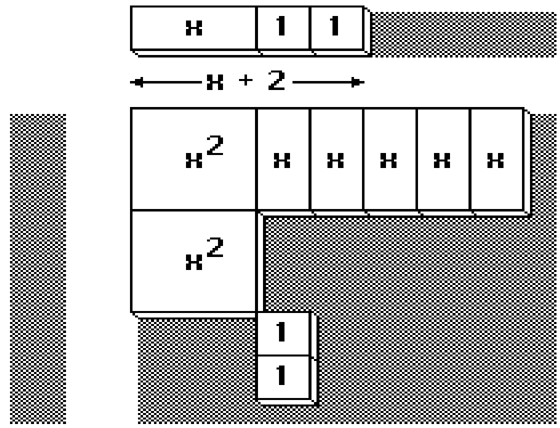
Division of Polynomials

To divide an expression by another, turn off the 'Stay Equal' check box, place the tiles you want to divide by (the divisor) in one of the guide bars, and then place the tiles for the expression you want to divide (the dividend) into the product area. Rearrange the tiles into a rectangle with one side matching the guide tiles. Now fill in the missing guide yourself or choose 'Stay Equal'.

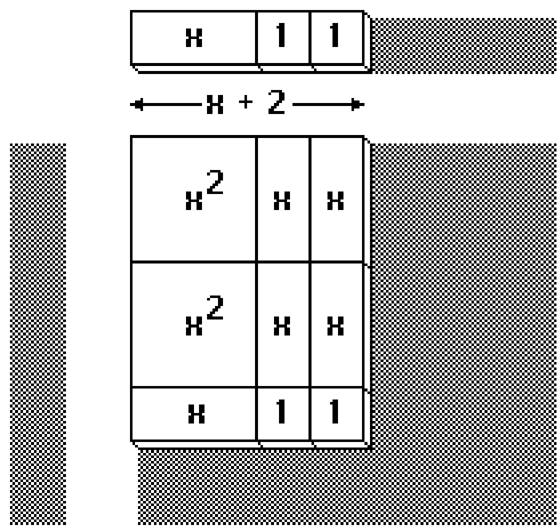
Example 1

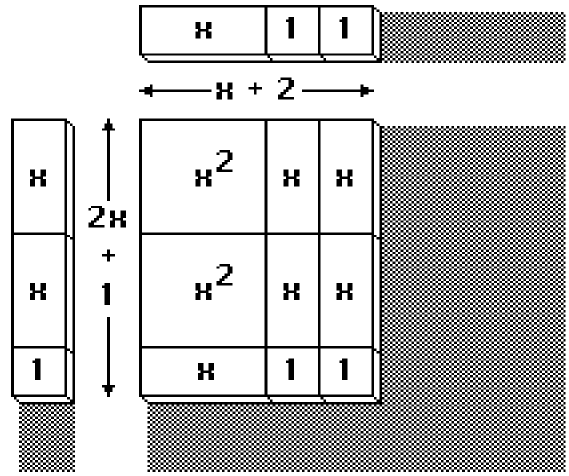
Turn off the 'Stay Equal' check box.

- To divide $2x^2 + 5x + 2$ by $x + 2$, drop the tiles for $2x^2 + 5x + 2$ into the product area, and then the tiles for $x + 2$ into the width bar (as shown).



- Rearrange the tiles for $2x^2 + 5x + 2$ so that they form a rectangle with width $x + 2$.



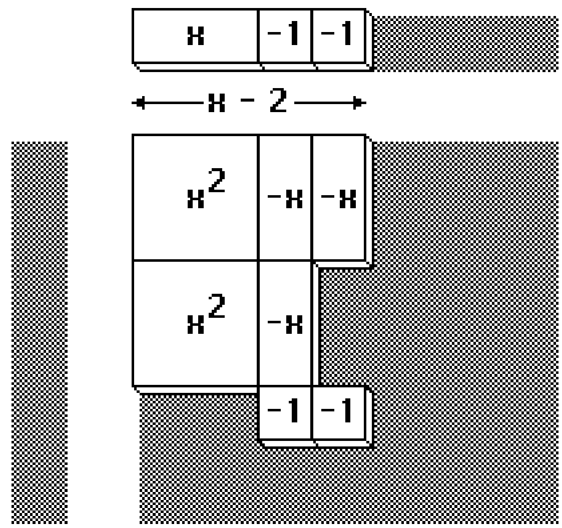


- Drag the tiles which represent the height of the rectangle into the height bar, click on the 'Stay Equal' check box. This represents the answer when $2x^2 + 5x + 2$ is divided by $x + 2$.

If you click on the 'Division by Width' radio button, the following will appear on the screen:

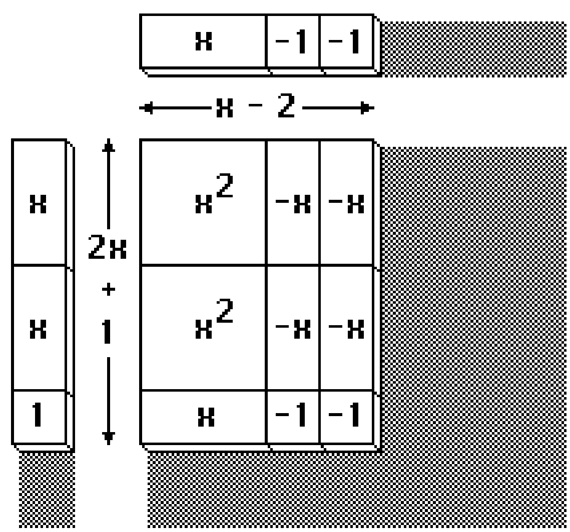
$$\frac{2x^2 + 5x + 2}{x + 2} = 2x + 1$$

Example 2



(Turn off the 'Stay Equal' box.) To divide $2x^2 - 3x - 2$ by $x - 2$, drop tiles into the appropriate areas, and then attempt to form into a rectangle with width $x - 2$.

The problem is that there aren't enough tiles ... so more have to be added. However, we can't just start adding tiles or we would change the question. If we add zero, we wouldn't change the expression, and we know that $-x$ plus x is zero, so...



- add $+x$ and $-x$ to the product area to complete the rectangle
- fill in the height bar or click on the 'Stay Equal' box to get $2x + 1$ as the correct answer.

If the 'Division by Width' radio button is blackened, the computer will display the following:

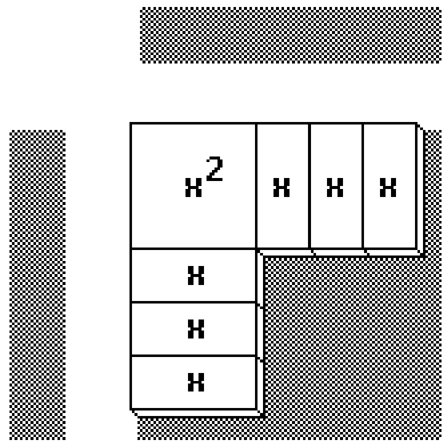
$$\frac{2x^2 - 3x - 2}{x - 2} = 2x + 1$$

Completing the Square

To complete the square using *Virtual Tiles*, just add in terms that would make a perfect square.

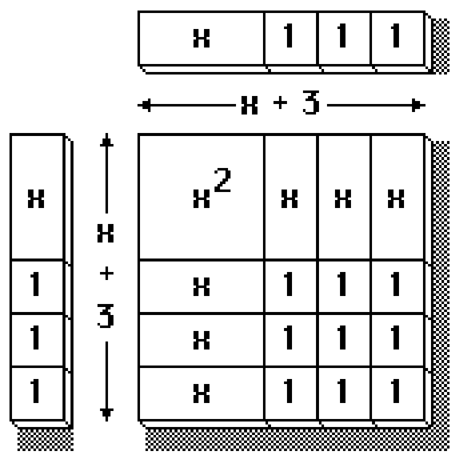
Example 1

Turn off the 'Stay Equal' box and 'Erase'. To complete the square using $x^2 + 6x$, fill in the product area and arrange the tiles so that the width and height of the rectangle will be the same (put the same number of x's on both sides).



Now

- fill in the missing pieces to 'complete the square'. You have just changed the question, so remember that you have a debt of 9 (because you added 9 tiles to the product area),
- fill in the guide tiles, and
- write the equivalent expression.



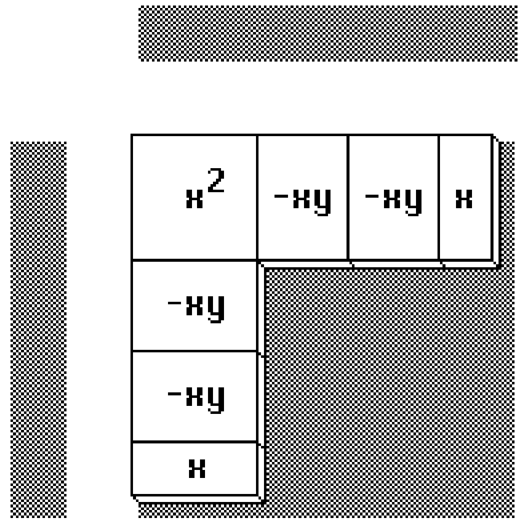
In this case,

$$x^2 + 6x$$
$$= (x^2 + 6x + 9) - 9$$
$$= (x + 3)^2 - 9$$

Example 2

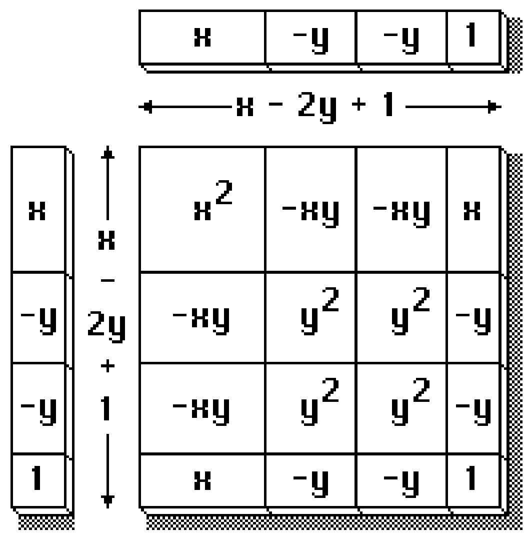
Students would rarely attempt problems like this one using paper and pencil, but it is a fairly simple extension using *Virtual Tiles*. Complete the square: $x^2 - 4xy + 2x$

As in the previous example, drag over the appropriate tiles and arrange the tiles so that the width and height of the rectangle will be the same.



Now

- fill in the missing pieces to 'complete the square'. You have changed the question by adding $4y^2 - 4y + 1$ tiles to the product area to complete the square.
- fill in the guide tiles, and
- write the equivalent expression.



In this case,

$x^2 - 4xy + 2x$
$= (x^2 - 4xy + 2x + 4y^2 - 4y + 1) - 4y^2 + 4y - 1$
$= (x - 2y + 1)^2 - 4y^2 + 4y - 1$

Factoring Polynomials

When you factor an expression, you must recognize that there are often many different ways to express an expression as the product of two or more factors.

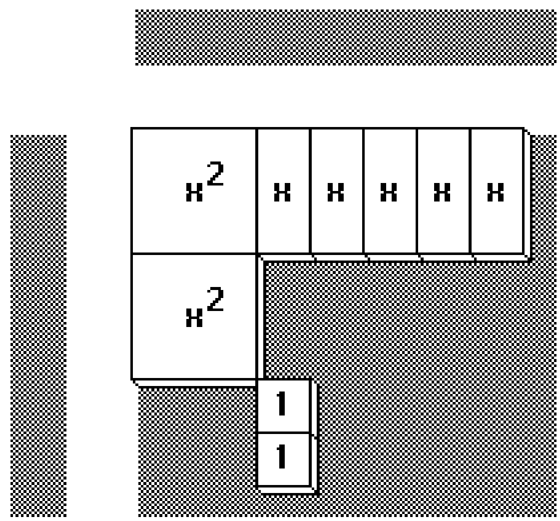
For example, in the positive integers, 12 can be expressed as 12 x 1, 6 x 2, 4 x 3, or 2 x 2 x 3. If we allow negative integers, then there are many other possible combinations. In the Rational or Real Number systems, there are clearly an infinite number of factorizations. *Virtual Tiles* will allow you to express a single polynomial as the product of two other expressions, simply by arranging the product tiles into a rectangle. We will assume that we are working with integral coefficients.

Example 1

Turn off the 'Stay Equal' check box.

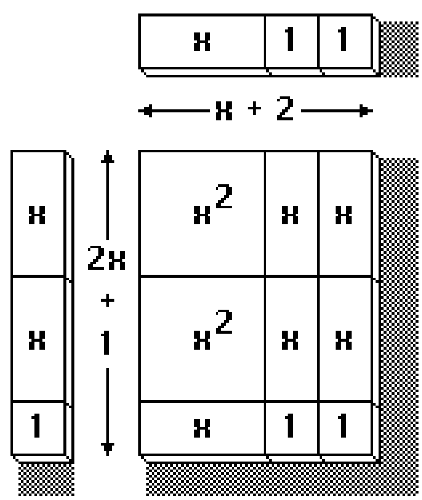
To factor $2x^2 + 5x + 2$:

- Drop the tiles for $2x^2 + 5x + 2$ into the product area (as shown).
- Rearrange the tiles into a rectangle. (Hint: arrange the x^2 tiles into a rectangle in the upper left and the 1's into a rectangle in the bottom right of the product area. If there were y^2 tiles, you would make sure they formed a rectangle as well.)



- Fill in the guide tiles, and write your answer. (You can choose 'Fill in Missing Guide Tiles' from the options menu if you would rather have the computer do the work!)

The answer is $(2x + 1)(x + 2)$

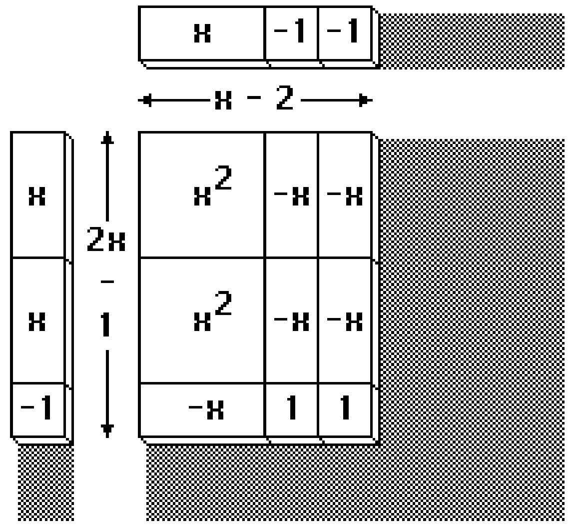


- Make sure the 'Stay Equal' check box is on. Now flip all of the horizontal and vertical guide tiles (double–click on them). This shows that $2x^2 + 5x + 2$ could also be expressed as $(-2x - 1)(-x - 2)$

Example 2

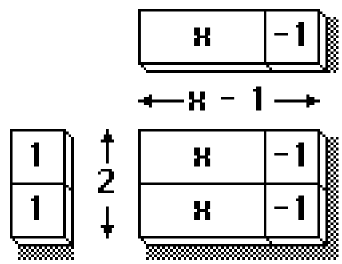
Factor $2x^2 - 6x + 4$

One possible factorization is shown on the right. While arranging terms it is important to pay attention to signs (think about which guide tiles would need to be flipped).



$$\text{i.e., } 2x^2 - 6x + 4$$
$$= (2x - 2)(x - 2)$$

However, students should be encouraged to look for factors within each factor. In this case, $2x - 2$ can be factored.
Students should be encouraged to discover this for themselves... as shown on the right.



Since $2x - 2 = 2(x - 1)$ then

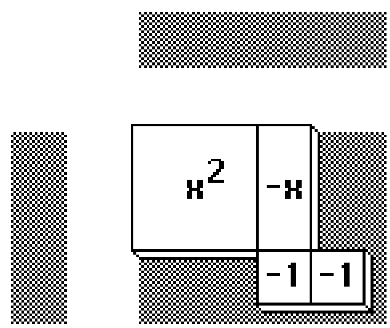
$$2x^2 - 6x + 4$$
$$= (2x - 2)(x - 2)$$
$$= 2(x - 1)(x - 2)$$

There are several different ways to arrive at the same solution!

Example 3

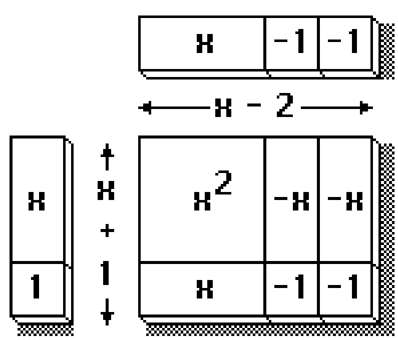
Factor $x^2 - x - 2$

Once those tiles are dropped into the product area, it is clear that a rectangle can't be formed without adding tiles (and you can only add zero to it). So ...



Add $+x$ and $-x$ to the expression to leave it unchanged and to complete the rectangle as shown.

$$\text{i.e. } x^2 - x - 2$$
$$= (x + 1)(x - 2)$$



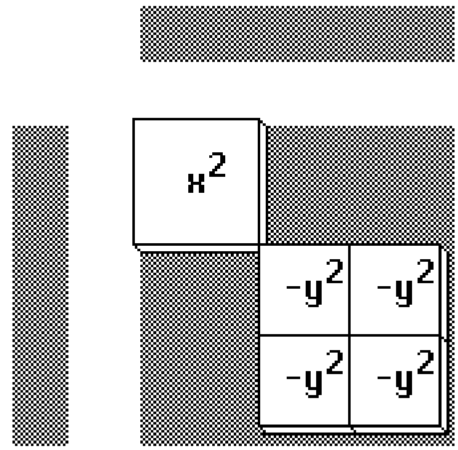
If you have selected 'Stay Equal', *Virtual Tiles* will fill in needed tiles to complete rectangles. HOWEVER, the result may be product tiles which are no longer equivalent to the original question! Be careful ... you may have to flip or remove tiles to get the right expression.

Example 4 (Difference of squares)

Factor $x^2 - 4y^2$

Once you have dropped in these tiles, a few things become obvious.

- both groups of terms form squares (hence difference of squares)
- in order to make this into a rectangle, we will have to add tiles to the diagram (that add to 0)

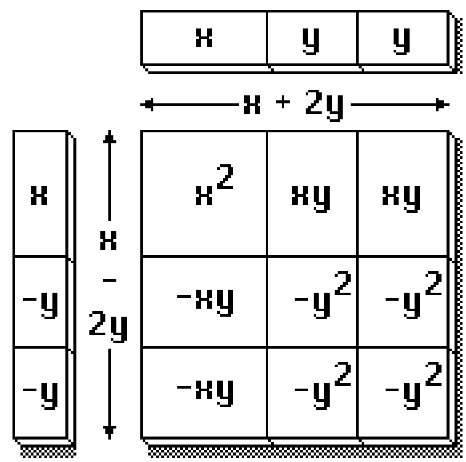


With a little experimenting, we find that we have to add two xy terms and two $-xy$ tiles to complete the rectangle.

The result is that we discover that

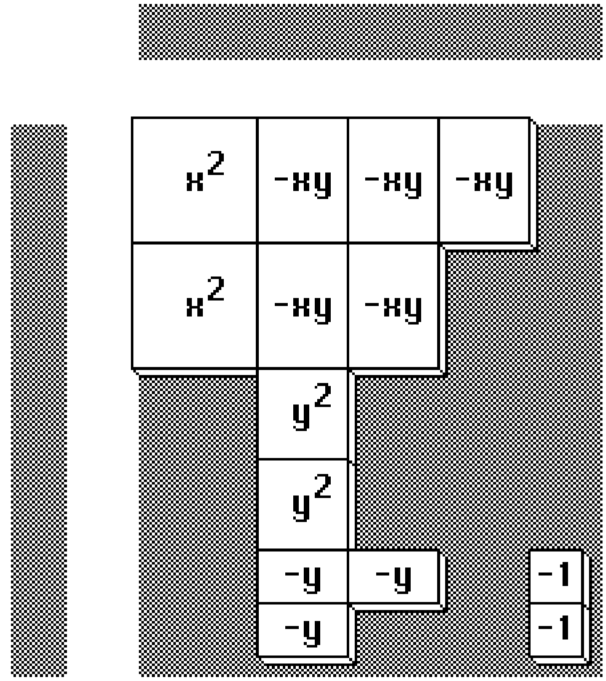
$x^2 - 4y^2$
$= (x - 2y)(x + 2y)$

If we attempt to factor $x^2 + 4y^2$, we discover that we would have to drop in the same tiles as before, but the signs just won't work out for the guide tiles.



Example 5

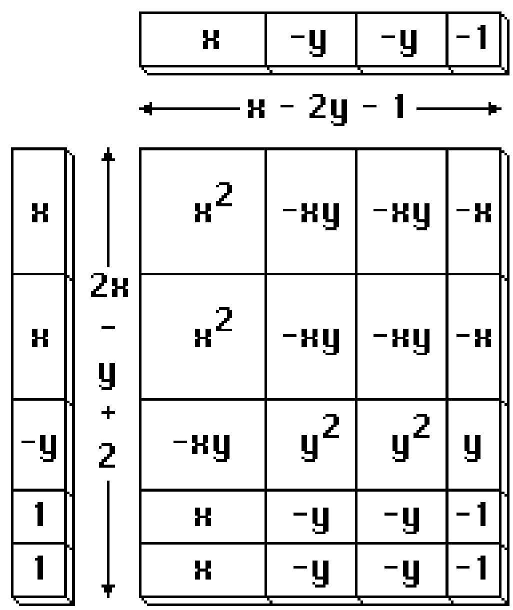
The challenge....
Factor $2x^2 - 5xy + 2y^2 - 3y - 2$



Drop the tiles in ...

- place the two x^2 tiles, and then don't move them.
- place the two y^2 tiles. There are two different arrangements for these tiles. If this arrangement doesn't 'work', try changing the position of these tiles.
- place the two –1 tiles. There are two different arrangements for these tiles as well. You can try rearranging these tiles as well to find the solution.
  (Note that these tiles can never be beside each other because the sides of each square are different from the others.)
- Now try placing the other tiles. It should be clear before long that this arrangement of tiles won't work... keep trying ... experiment with 'Stay Equal'.

The solution is shown on the right. Try solving this type of question without *Virtual Tiles*!



Experiment with other expressions.

Student Practice Sheet

Name:______________________________
Date: ______________________________

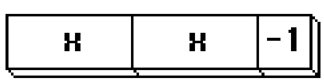
Introduction to the Distributive Property

Using instructions from your teacher, turn on the computer, and open *the Virtual Tiles* program (by double clicking on the icon).

So far, you have learned how to simplify expressions by combining like terms. This only works in expressions where the terms are separated by addition or subtraction. So... you should be able to simplify things like $-2x^2 - 3 + 5xy + y - xy + x^2 + 2$. By the way, the answer is $-x^2 - 1 + 4xy + y$. These expressions are equal because no matter what numbers we substitute for x and y, both expressions would give the same answer.

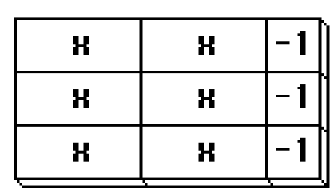
However, you probably don't know how to simplify expressions like $3(2x - 1)$. Normally, using the order of operations this would mean you would have to work out 2 x x subtract 1, and then take that answer and multiply by 3. We are trying to find another expression that wouldn't have any brackets in it that would give the same answer no matter what number x was.

First, $2x - 1$ can be represented by,



so 3 times $(2x - 1)$ just means we should repeat this 3 times.

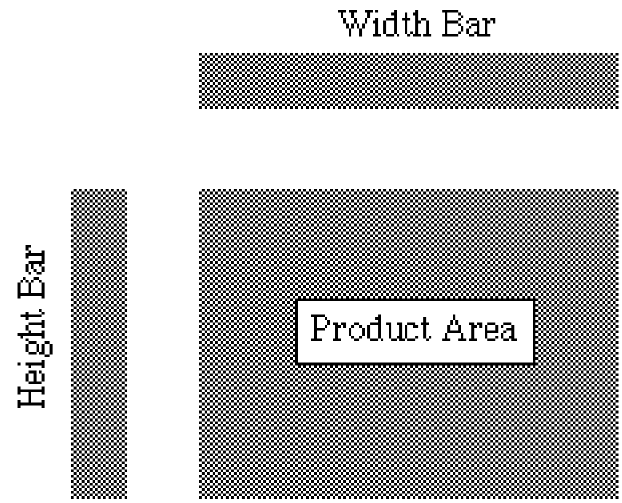
So, $3(2x - 1)$ is represented by...



The answer is $6x - 3$

In *Virtual Tiles*, select 'Multiplication' from the 'Environment' menu. Click on the 'Stay Equal' check box.

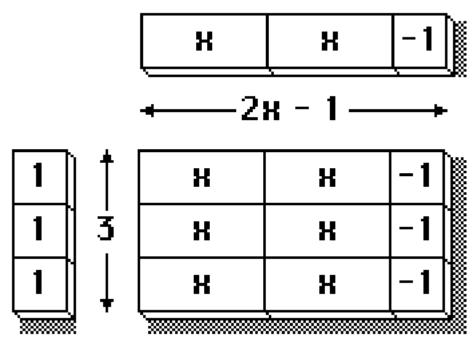
You should see something like this on the screen:



As you drag tiles into the height and width bars, the computer will fill in tiles in the product area. The tiles you drag into the width and height bars are called the 'guide tiles'. Can you figure out what's happening?

See if you can get 3(2x – 1) to appear in the product area. Hint... try to get the product area to look like the picture above.

One way to do this is to drag 3 into the height column and 2x – 1 into the width column as shown on the right.



The diagram shows that 3(2x – 1) = 6x – 3.

So... when you want to multiply two expressions, turn on the 'stay equal' check box, then drag the tiles into the height and width bars. The answer will appear in the product area.

Questions

1. Start with the diagram above. Flip one of the 1–tiles in the height bar by double–clicking on it.
   a) What happened to each of the tiles in the height bar?
   b) What happened to the tiles in the product area?
   c) This shows that $-3(2x - 1) =$ ____________.

   d) $-3(2x + 1) =$ ____________
   e) $-3(-2x + 1) =$ ____________
   f) $3(-2x + 1) =$ ____________
   g) $3(-2x - 1) =$ ____________

2. Simplify

   a) $2(x + y) =$ ____________
   b) $2(x - y) =$ ____________
   c) $-2(x - y) =$ ____________
   d) $-2(x + y) =$ ____________
   e) $-2(-x + y) =$ ____________
   f) $2(x + y + 1) =$ ____________
   g) $2(x + y - 1) =$ ____________

3. a) $-(x + y)$ means -1 times $(x + y)$. therefore $-(x + y) =$ ____________

   b) $-(x - y) =$ ____________
   c) $-(-x + y) =$ ____________
   d) $-(2x + 1) =$ ____________
   e) $-(2x - 1) =$ ____________
   f) $-(2x + y + 1) =$ ____________
   g) $-(2x - y + 1) =$ ____________
   h) $-(2x - y - 1) =$ ____________
   i) $-(-2x + y - 2) =$ ____________

4. Simplify

   a) $x(x + 1) =$ ____________
   b) $x(x - 1) =$ ____________
   c) $x(x + 2) =$ ____________
   d) $x(x - 2) =$ ____________
   e) $x(2x + 1) =$ ____________
   f) $x(2x - 1) =$ ____________
   g) $2x(x + 1) =$ ____________
   h) $2x(x - 1) =$ ____________
   i) $2x(x + 2) =$ ____________
   j) $x(x + y + 1) =$ ____________

5. a) Simplify $(x + 3)(x - 3y + 2) =$ ______________________

   b) Make up your own question and simplify. Be creative!!

Student Practice Sheet

Name:________________________

Date: ________________________

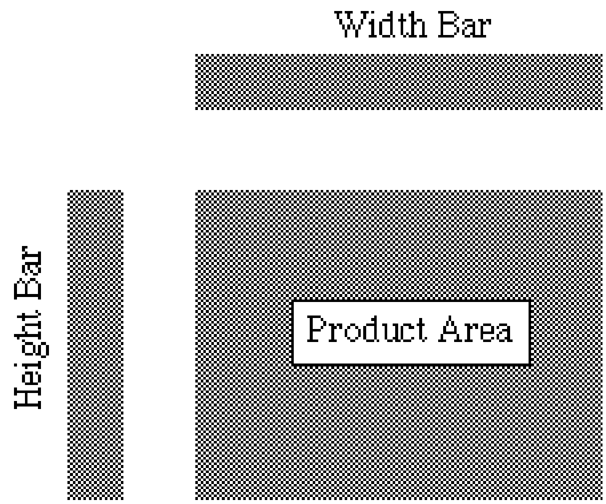
Introduction to the Multiplication of Polynomials

Using instructions from your teacher, turn on the computer, and open the *Virtual Tiles* program (by double clicking on the icon).

You have learned how to simplify expressions like 3(2x – 1) by expanding using the distributive property. i.e., you should know that this is equivalent to 6x – 3 because the 3 must be multiplied by both the 2x and the –1. We will now check this using *Virtual Tiles*.

Select '**Multiplication**' under the 'Environment' menu and turn on the '**Stay Equal**' check box.

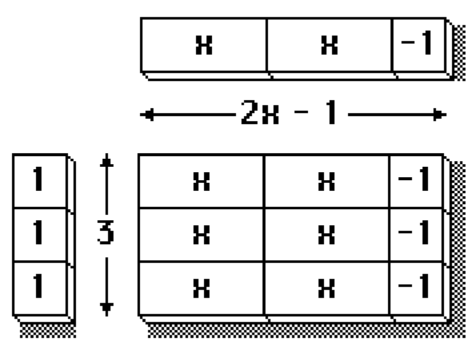
You should see something like this on the screen:



As you drag tiles into the height and width bars, the computer will fill in tiles in the product area. The tiles you drag into the width and height bars are called the 'guide tiles'. Can you figure out what's happening?

See if you can get 3(2x – 1) to appear in the product area. Hint... try to get the product area to look like the picture above.

Hopefully, you dragged 3 into the height column and 2x – 1 into the width column as shown on the right.

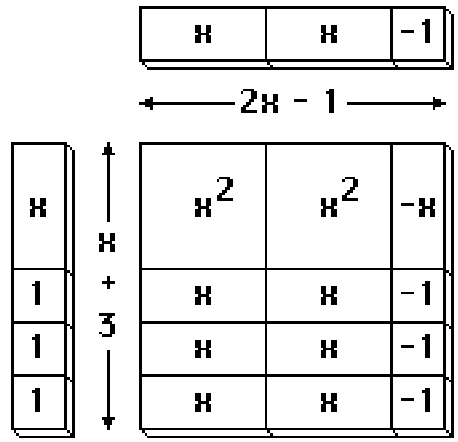


The diagram shows that 3(2x – 1) = 6x – 3.

So... when you want to multiply two expressions turn on the 'stay equal' check box then drag the tiles into the height and width bars. The answer will appear in the product area.

You can use this technique to multiply more complicated expressions.

Example



Simplify (x + 3)(2x – 1)
Because the brackets are around the x + 3 and the 2x – 1, it means we must multiply all of x + 3 by all of 2x – 1.

(If the question had been x + 3(2x – 1), only the 3 would be multiplied by the 2x – 1 and then x added afterwards because of order of operations.)
To do this, drag x and three 1–tiles into the height bar. Drag 2x – 1 into the width bar. The answer is the collection of tiles in the product area (as shown on the right).

The diagram shows that (x + 3)(2x – 1) $= 2x^2 - x + 6x - 3$
This simplifies to become $= 2x^2 + 5x - 3$

Questions

1. Start with the diagram above.
   a) What does x(2x – 1) equal? Where does this answer appear in the diagram? Why?

   b) What does 3(2x – 1) equal? Where does this answer appear in the diagram? Why?

   c) The diagram shows that (x + 3)(2x – 1) = x(2x – 1) + 3(2x – 1). Explain.

   d) Flip the three 1–tiles in the height bar. What effect does this have on the tiles in the product area?

   e) This shows that (x – 3)(2x – 1) = _______________

   f) Flip the 1 tile in the width bar. What effect does this have on the tiles in the product area?

   g) This shows that (x – 3)(2x + 1) = _______________

2. Use *Virtual Tiles* to expand and simplify each of the following:

   a) (–x + 1)(2x + 3) = ________________

   b) (x + 1)(2x + 3) = ________________

   c) (x – 1)(2x + 3) = ________________

d) $(x - 1)(2x - 3) =$ _______________

e) In each case, how could you predict the number of x^2 tiles in the answer?

f) In each case, how could you predict the number of 1 tiles in the answer?

g) In each case, how could you predict the number of x–tiles in the answer?

3. Use *Virtual Tiles* to expand and simplify each of the following:

a) $(x + 1)(x - 1) =$ _______________

b) $(x + 2)(x - 2) =$ _______________

c) $(2x + 1)(2x - 1) =$ _______________

d) $(2x - 3)(2x + 3) =$ _______________

e) What patterns do you notice in each of the questions above? What happens in each case when you simplify?

f) In each case, how could you predict the number of x^2 tiles in the answer?

g) In each case, how could you predict the number of 1–tiles in the answer?

h) In each case, how could you predict the number of x–tiles in the answer?

4. You know that $(x + 3)^2$ means $(x + 3)(x + 3)$. Use this idea and *Virtual Tiles* to expand and simplify each of the following:

a) $(x + 3)^2 =$ _______________

b) $(x - 3)^2 =$ _______________

c) $(x + 1)^2 =$ _______________

d) $(2x + 3)^2 =$ _______________

e) What shape do the tiles form in the product area for each question above?

f) In each case, how could you predict the number of x^2 tiles in the answer?

g) In each case, how could you predict the number of 1 tiles in the answer?

h) In each case, how could you predict the number of x tiles in the answer?

5. Use *Virtual Tiles* to expand and simplify each of the following:

   a) $(x + y)(2x + 3y) =$ ________________

   b) $(x + y)(2x - 3y) =$ ________________

   c) $(x - y)(2x - 3y) =$ ________________

   d) $(x - y)(2x + 3y) =$ ________________

   e) In each case, how could you predict the number of x^2 tiles in the answer?

   f) In each case, how could you predict the number of y^2 tiles in the answer?

   g) In each case, how could you predict the number of xy tiles in the answer?

6. Use *Virtual Tiles* to expand and simplify each of the following:

   a) $(x + y + 1)(2x + 3y - 1) =$ ________________

   b) $(2x - y + 1)(2x - y + 3) =$ ________________

   c) $(2x - 3y - 3)(2x - 3y + 3) =$ _____________

   d) $(x - y + 2)(x + y - 2) =$ _________________

7. Make up your own question and simplify. Be creative!

Student Practice Sheet

Name:___________________________
Date: ___________________________

Introduction to Factoring

You have learned how to simplify expressions like -2y(3x - y) or (x + 3)(2x – 1) by expanding and then collecting like terms. You will now learn how to use *Virtual Tiles* to reverse this process ... to factor. When you are factoring an expression, you are trying to express it as two other expressions multiplied. It's like a certain game show where they give you the answer and you have to guess the question. So if you are asked to factor $x^2 + 3x$, you should guess x(x + 3). *Virtual Tiles* makes it much easier to do this.

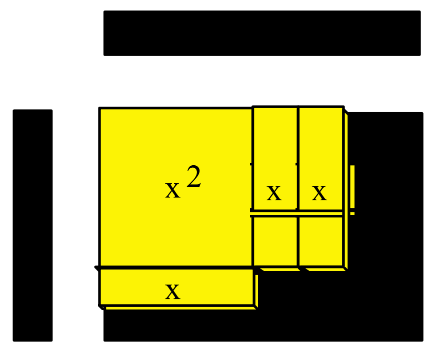
Using instructions from your teacher, turn on the computer, and open the *Virtual Tiles* program (by double clicking on the icon).

Select '**Multiplication**' under the 'Environment' menu.

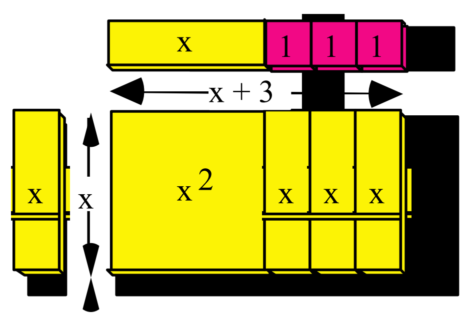
Example 1

Make sure the 'Stay Equal' check box is not checked.
To factor $x^2 + 3x$:

- Drag the tiles for $x^2 + 3x$ into the product area
- Rearrange the tiles into a rectangle.



- Click on the 'Stay Equal' box, and the computer will fill in the guide tiles for you.



This shows that $x^2 + 3x = x(x + 3)$

Practice

1. Factor each of the following:

a) $x^2 - 3x$ = ____________________

b) $2x + 4$ = ____________________

c) $2x^2 - 4x$ = ____________________

d) $3x^2 + 3x$ = ____________________

e) $xy - 3x$ = ____________________

f) $y^2 + xy$ = ____________________

g) $x^2 + 2x + 1$ = ____________________

h) $x^2 - 2x + 1$ = ____________________

i) $x^2 + 3x + 2$ = ____________________

j) $x^2 + 4x + 3$ = ____________________

k) $x^2 + 5x + 4$ = ____________________

l) $x^2 + 4x + 4$ = ____________________

m) $x^2 + 6x + 5$ = ____________________

n) $x^2 + 5x + 6$ = ____________________

o) $x^2 + 7x + 6$ = ____________________

p) $x^2 + 6x + 8$ = ____________________

q) $x^2 + 9x + 8$= ____________________

r) $x^2 + 6x + 9$ = ____________________

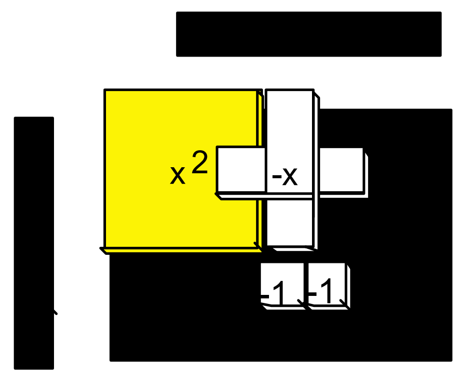
s) $x^2 + 7x + 12$ = ____________________

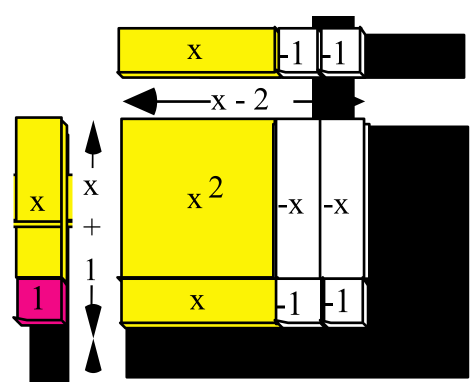
t) $x^2 + 8x + 12$ = ____________________

u) $x^2 + 3xy + 2y^2$ = ____________________

u) $x^2 + 5xy + 6y^2$ = ____________________

v) $x^2 + 3xy + 2y^2 + 2x + 3y + 1$ __________





Now that you have done this, rearrange the tiles into a rectangle as shown, and click on the 'Stay Equal' check box.
This shows that

$$x^2 - x - 2$$
$$= (x + 1)(x - 2)$$

To do other questions like this, try to rearrange the tiles into a rectangle without adding anything first, and then be careful to add one positive and one negative tile at a time.

Practice

1. Factor each of the following:

a) $x^2 + x - 2 =$ ________________

b) $x^2 + 2x - 3 =$ ________________

c) $x^2 - 2x - 3 =$ ________________

d) $x^2 - 4 =$ ________________

e) $x^2 + 3x - 4 =$ ________________

f) $x^2 - 4x - 5 =$ ________________

g) $x^2 + x - 6 =$ ________________

h) $x^2 - x - 6 =$ ________________

i) $x^2 + 5x - 6 =$ ________________

j) $x^2 - 9 =$ ________________

k) $x^2 + 2x - 8 =$ ________________

l) $y^2 - 4y - 12 =$ ________________

m) $x^2 + xy - 6y^2 =$ ________________

n) $x^2 + 5xy - 6y^2 =$ ________________

o) $x^2 - xy - 6y^2 =$ ________________

p) $x^2 - 9y^2 =$ ________________

q) $4x^2 - y^2 =$ ________________

r) $6x^2 - x - 1 =$ ________________

s) $2x^2 - 5x - 12 =$ ________________

t) $x^2 - 6x + 9 =$ ________________

u) $x^2 - 6x + 9 - y^2 =$ ________________

v) $4 - x^2 + 4xy - 4y^2 =$ ________________

w) $x^2 - y^2 + x - 3y - 2 =$ ________________