MATH TREK High School

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NECTAR Foundation

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Authorware was used to create *Math Trek High School*.

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OVERVIEW

Introduction

Math Trek High School is an integrated, curriculum-based, Mathematics program for high school students. It is an engaging multimedia program with sound, animation, and graphics, which provides direct links to the curriculum. The program is organized by the typical strands found in the curricula of most educational jurisdictions. Tutorials, activities, performance tasks, and on-line assessment provide multiple contexts and opportunities for students to demonstrate their knowledge and skills. The program may be used on standalone computers or on a network, on either Macintosh or Windows computers.

Curriculum Content

Math Trek High School covers the standard curriculum generally taught in high school. It follows the philosophy and guidelines contained in the NCTM (National Council of Teachers of Mathematics) and OAME (Ontario Association of Mathematics Educators), the grades 9-12 curriculum expectations in the new Ontario Curriculum for mathematics; and the specific and general objectives in the Common Curriculum Framework for Mathematics (Western and Northern Canadian Protocol for Mathematics).

Program Components

The program components include:

- Interactive Tutorials
- Practice and Teacher Configurable Quizzes
- Mathematics Glossary
- Student Tracking System
- Teacher Support Materials

There are fourteen major content sections in *Math Trek High School*:

- Numbers
- Polynomials
- Equations
- Geometric Relationships
- Statistical Relationships
- Linear Functions
- Linear Systems
- Factoring
- Quadratic Equations
- Quadratic Functions
- Inequalities
- Linear Inequalities
- Function
- Rational Expressions

Organization and Structure

This chart delineates the program sections for each topic. A quiz concludes each of the sections.

Topic	Sections
Numbers	1. Integers 2. Rational Numbers 3. Squares and Square Roots 4. Evaluating Powers 5. Exponent Laws 6. Negative/zero Exponents 7. Scientific Notation
Polynomials	1. Introduction 2. Adding and Subtracting 3. Multiplying Polynomials
Equations	1. Introduction 2. Solving Equations 3. Formulas
Geometric Relationships	1. Angles 2. Triangles 3. Polygons 4. Surface Area 5. Volume
Statistical Relationships	1. Collecting Data 2. Organizing Data 3. Analyzing Data 4. Line of Best Fit
Linear Functions	1. Introduction 2. Linear and Non-linear Relationships 3. Slope and Intercepts 4. Graphing Slope/Intercept Linear Equations 5. Alternate Forms of Linear Equations 6. Interpolation and Extrapolation 7. Applications
Linear Systems	1. Introduction 2. Graphs 3. Substitution 4. Elimination 5. Applications
Factoring	1. Introduction 2. Greatest Common Factor 3. Difference of Squares 4. Perfect Squares 5. Simple Trinomials 6. Complex Trinomials 7. Multi-step Factoring
Quadratic Equations	1. Introduction 2. Solving by Isolation 3. Solving by Factoring 4. Solving by Completing the Square 5. Quadratic Formula 6. Isolation
Quadratic Functions	1. Introduction 2. Graphing

	3. Curve of Best Fit 4. Applications
Inequalities	1. Introduction 2. Solving Inequalities
Linear Inequalities	1. Graphs 2. Systems 3. Applications
Functions	1. Introduction 2. Domain 3. Range 4. Values and Zeros
Rational Expressions	1. Introduction 2. Stating Restrictions 3. Simplifying 4. Multiplying 5. Dividing 6. Adding and Subtracting

Technical Support

The NECTAR Foundation is committed to its customers. Technical support is provided at no charge to our registered customers. Answers to commonly asked questions can be found in the Technical Support section of the NECTAR web site www.nectar.ca.

You can reach NECTAR Foundation Technical Support via phone, e-mail or web page.

Technical Support Phone Number
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E-mail: support@nectar.ca Web: www.nectar.ca

GETTING STARTED

Check Requirements

Windows: Microsoft Windows 95 or later a Pentium, and 305 MB of hard disk space.

Macintosh: MAC OS 8.6 or later, or MAC OS X, and 310 MB of hard disk space.

FEATURES

Automatic Audio - Audio in *Math Trek High School* may be turned ON or OFF by teachers or students. This option is available when logging in to the program or via the Tools button.

Tools - Four items are available by clicking on the Tools Button. They are:

- Calculator;
- Journal;
- Glossary;
- Automatic Audio.

Quizzes and Practice - For each strand the following are provided:

(a) Practice questions – The student selects the question types. Questions are randomly generated and marked by the program. “Show Answer” and “Explain” are available.
(b) Sample Quiz - Several questions are provided in each sample quiz. The quiz is scored and results are stored in the student’s file.
(c) Teacher Quiz – Teachers may create their own quiz and include any number and type of questions. This is done through the teacher utility accessed by logging in as a teacher.
(d) Quiz Results – For each quiz or quiz subsection the following feedback is available to the teacher:

- question results;
- mean;
- median;
- standard deviation;
- number of students taking the quiz.
- results may be printed

OTHER FEATURES

- multimedia with audio instructions;
- interactive activities;
- student tracking system;
- comprehensive mathematics content;
- quizzes and interactive activities with immediate feedback;
- sound, graphics, and animation;
- off-screen performance tasks for individual and small group work;
- curriculum planning materials;
- integrated assessment instruments such as checklists, scoring rubrics and quizzes.

Install the Software

Windows: Insert the CD into your CD drive. Select Run from the Start menu. D:\Setup (or other letter identifying your CD-ROM drive) appears. Select OK and follow instructions. If installing onto a network, be sure to select a mapped network drive when prompted for the install location.

Macintosh: Insert the CD into your CD drive. Drag and drop the folder called Math Trek High School from the CD onto your hard drive. If installing onto a network, be sure to install onto your network hard drive.

Network Permissions

Once the installation is complete, assign proper permissions to the Math Trek High School folder. Network installation requires that users have write access to the folder called **Data**, located in the **Math Trek High School Xtras** folder. For all other Math Trek components, read-only access is sufficient. Please refer to your network software documentation.

Launch the Program

Follow these steps after installation.

Windows: **Go to** Start, Programs **select** NECTAR Foundation **and** Math Trek High School.

Macintosh: On your hard drive, double click the application file called **Math Trek High School Login** located in the folder where you have installed the software.

Login

Log in and enter Math Trek in one of these three ways:

a) Guest login: Click the Guest button. (Results for users logged in as Guests are **not** tracked.)

b) Student login: Select a class and click the name of a student in that class from the lists on screen. Initially, there will be one predefined class called **Sample** and two demo students called **Jean Morin and Leslie Park**. Click one of the demo students – a password is NOT required. Additional students and passwords are assigned to classes by the teacher. Results for users logged in as students are tracked.

c) Teacher login: Click the Teacher button. Enter the password **drowssap**. A user logged in as a Teacher has access to the Student Tracking System and can add classes and students, and change passwords. See the Teacher Utility section.

Access the Instructional Components

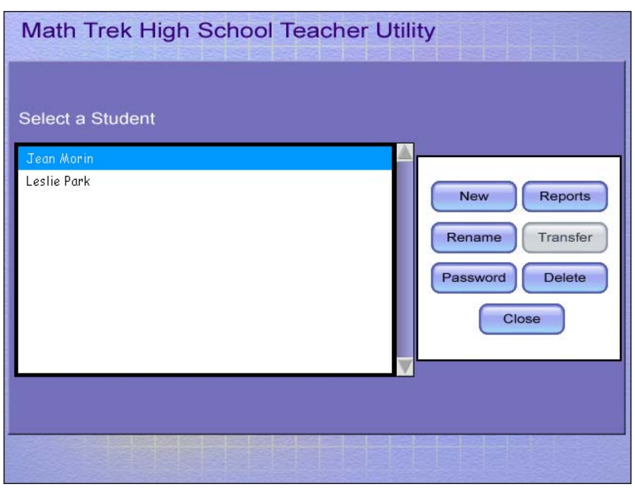
Select a Module

To access the instructional components, first login as a guest or as a student. Select a module from the opening screen. Click one of the modules. You can then choose a topic, then a subtopic.

Access the Teacher Utility

The Teacher Utility is used to track student activity and achievement, to add classes, students and passwords, and to print results.

To access the Teacher Utility log in as a Teacher. Select a class and click Open. See the instructions later in this manual for adding classes and students and for viewing and printing student results.



Teacher Utility Screen

INTERACTIVE TUTORIALS

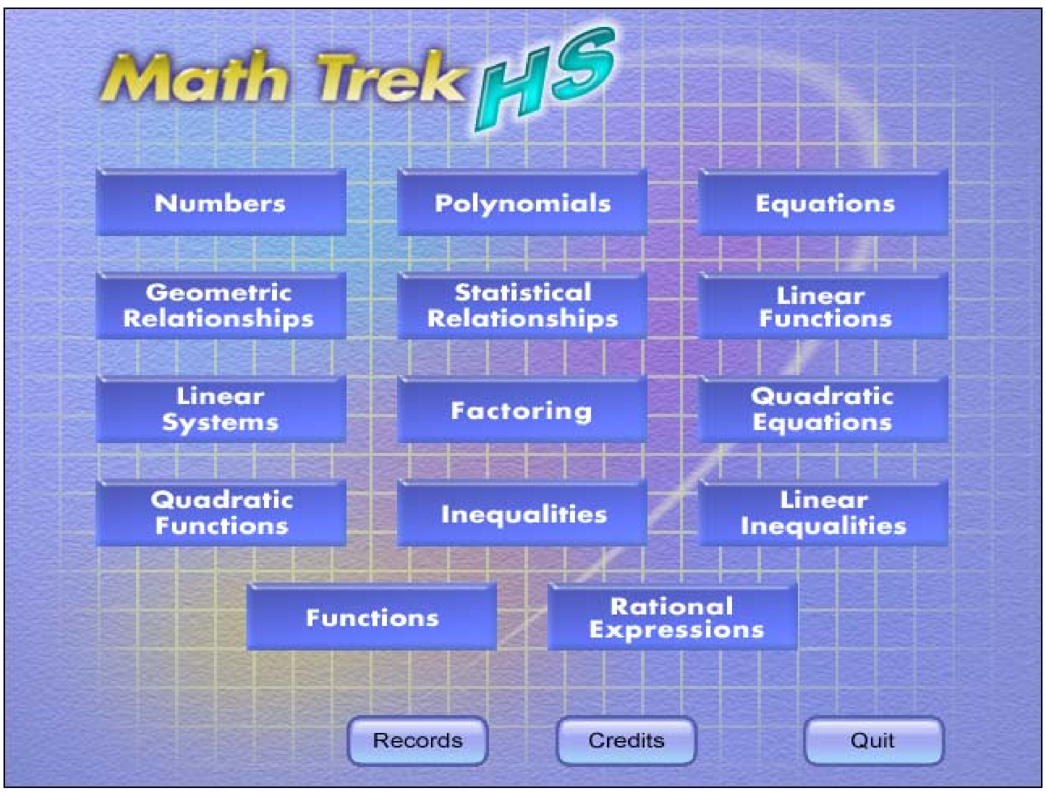
About the Tutorials

The tutorials are designed for self-paced learning by students. They:

- foster self-directed learning
- are often based on problem solving
- provide sample problems with step-by-step solutions
- make effective use of graphics, colour and animation
- provide immediate feedback
- supply randomly generated examples
- contain periodic review

Accessing the Tutorials

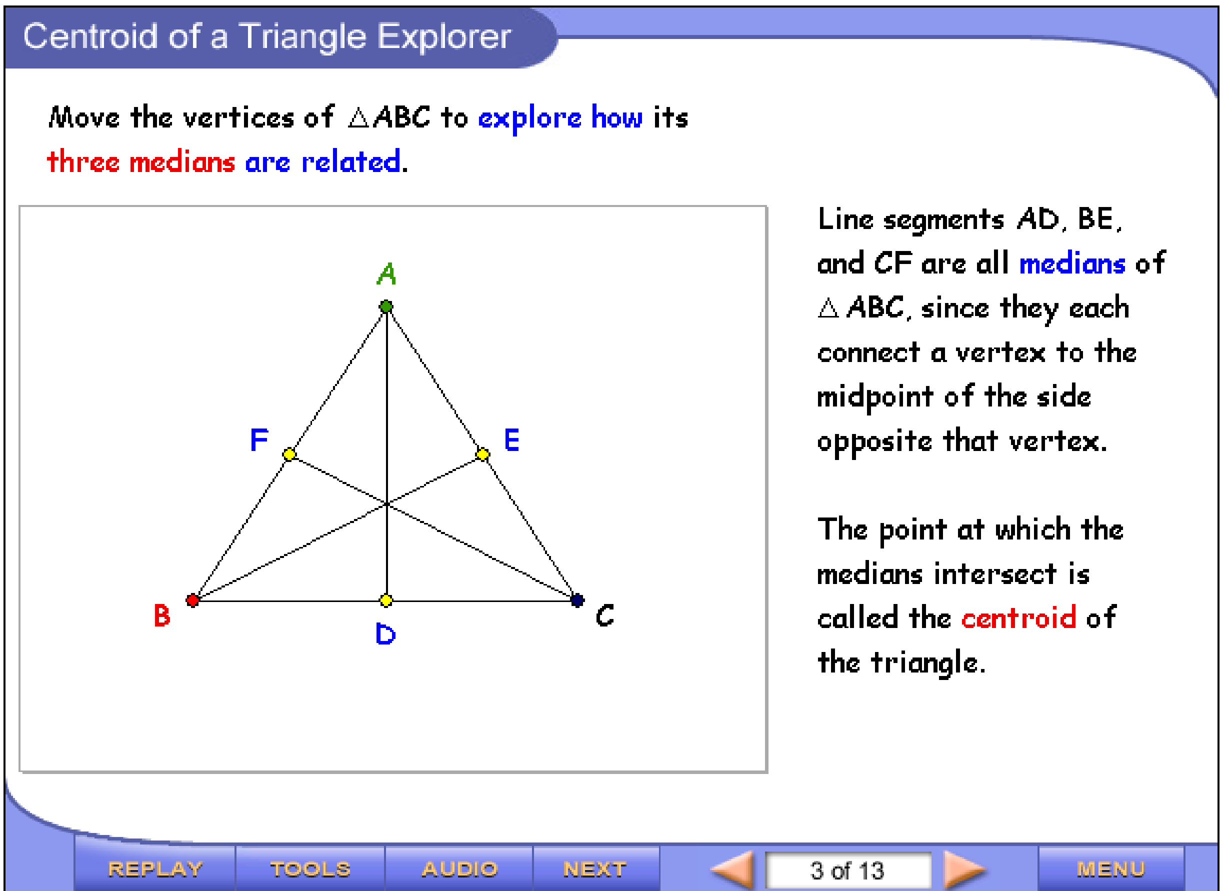
- Log in as a guest or student.
- Choose the strand from the main menu choices. Select a strand from the opening screen, (Numbers, Polynomials, Equations, Geometric Relationships, Statistical Relationships, Linear Functions, Linear Systems, Factoring, Quadratic Equations, Quadratic Functions, Inequalities, Linear Inequalities, Functions, and Rational Expressions). Click one of the strands. You can then choose a topic, then a subtopic.
- Select Interactive Tutorial from the choices in the rectangle below the strands.



Main Menu Screen

Tutorial Navigation Buttons

Each tutorial screen has a number of buttons along the bottom of the screen. Their functionality is detailed below.



Use this button to….

- REPLAY: … replay the current screen.
- TOOLS: ...provides access to a calculator, glossary and journal, as well as an option to play all audio automatically.
- AUDIO: … play the audio for most recently displayed text.
- NEXT: … display the next step of this screen.
- Left arrow: …return to the previous screen.
- Right arrow: … move to the next screen.
- MENU: … exit this part of the software.

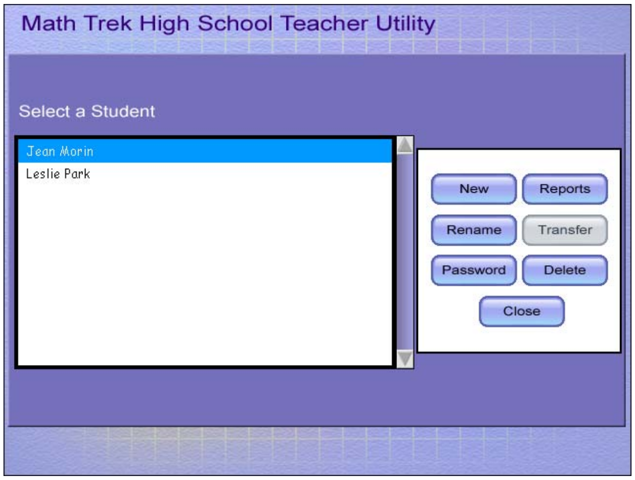
TEACHER UTILITY

About the Teacher Utility

The Teacher Utility is a student tracking and teacher management system. It is used to create and manage classes, set up student logins and view student activity and achievement.

Accessing the Teacher Utility

To access the Teacher Utility, launch Math Trek then click on the Teacher button on the login screen. Enter the teacher password, **drowssap.**



Managing Classes

The Math Trek Teacher Utility main menu screen will appear, listing all of the Math Trek classes.

- **To add a new class**, click the **New** button.
- **To delete a class**, select the class to delete then click the **Delete** button.
- **To rename a class**, select the class to rename then click the **Rename** button.
- **To create or change a class password**, select the class, then click the **Rename** button.
- **To see and manage the records for a particular class and its students**, select the class, and then click the **Open** button. (See the following section, Managing Students, for details.)
- **To leave the management system**, click the **Close** button.

Managing Students

To create and manage student logins and view student activity and achievement, log in as a teacher, select a class on the Teacher Utility main menu screen, then click the **Open** button. A Math Trek Class menu screen will appear, listing all of the students that have been created in the selected class.

- **To add a new student,** click the **New** button. Enter the student's name and password.

- **To add several new students from a text file**, click the **Import** button, find and select the file containing the student names, then click the **Open** button.

- **To delete a student,** select the student to delete, then click the **Delete** button.

- **To rename a student**, select the student to rename, then click the **Rename** button.

- **To transfer students to a different class**, click the **Transfer** button. Select the class you are transferring students to, then click **OK** button. Drag and drop the students to be transferred from the source class on the left to the destination class on the right. After moving all the students you wish to transfer, click the **Transfer** button.

- **To create or change a student password**, select the student, then click the **Rename** button.

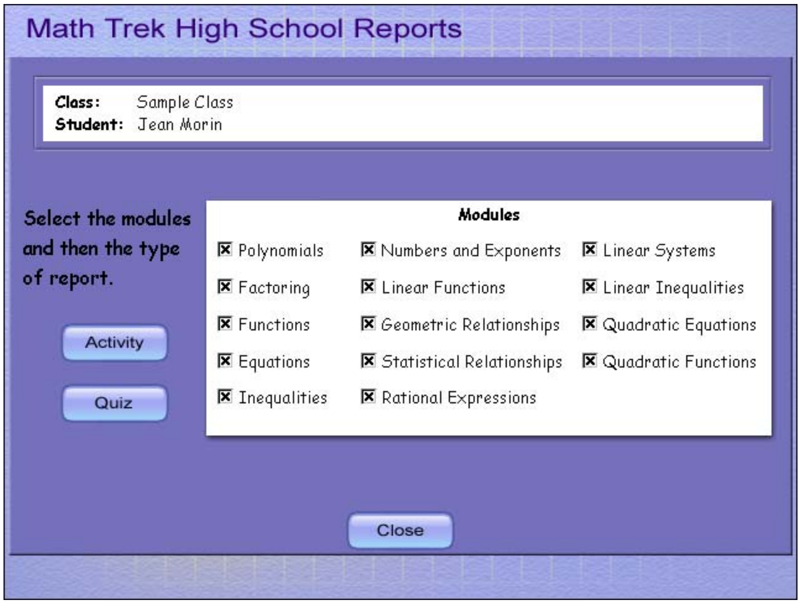
- **To view activity and achievement records for a student**, select the student, then click the **Reports** button. (See the following section, **Viewing any Student's Results and Activity as a Teacher**, for details)

- **To go back to the Teacher Utility main menu screen, click the Close button.**

Viewing any Student's Results and Activity as a Teacher

The Teacher Utility's Tracking System provides a record of each student's test results and a history of their activity with the software.

To view a record of any student's test results and a history of their activity with the software, log in as a teacher, select a class on the Math Trek Teacher Utility main menu screen, then click the **Open** button. Next, select a student on the Math Trek Class menu screen, then click the **Reports** button. The Math Trek Student Reports menu screen will appear. Click either the **Quiz Results** or **Activity Report** buttons.

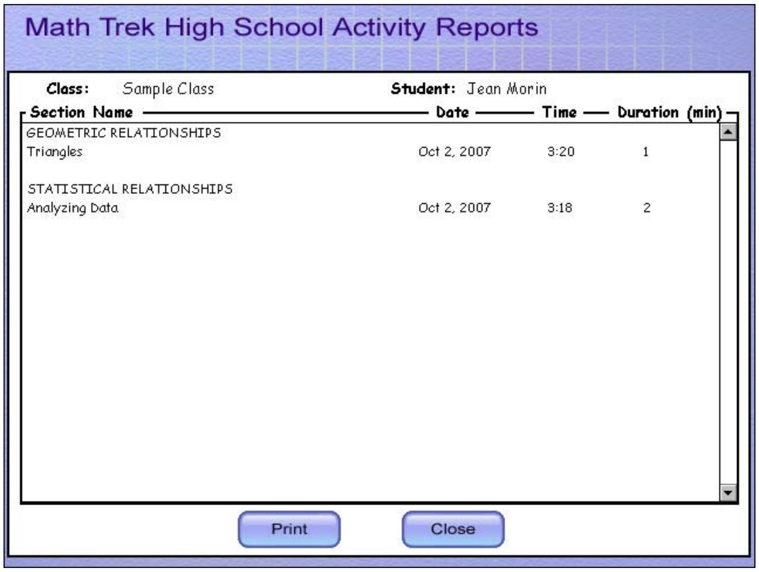


Math Trek Student Reports Menu Screen

Viewing Student Results and Activity as a Student

As a student, you may view your own test results and activity history. Log in as a student, then click the **Your Marks** button on the main menu screen. The Math Trek Student Reports menu screen will appear. Click either the **Quiz Results** or **Activity Report** buttons.

This is an activity report for student Jean Morin. It indicates the strand, date, time and length of time spent by Jean on particular exhibits.



A Student Activity Report

Changing the Teacher Utility Password

On installation, the default password is ***drowssap.*** Whenever you log in as a teacher, you may change the password so that it is unique to you. Follow these steps:

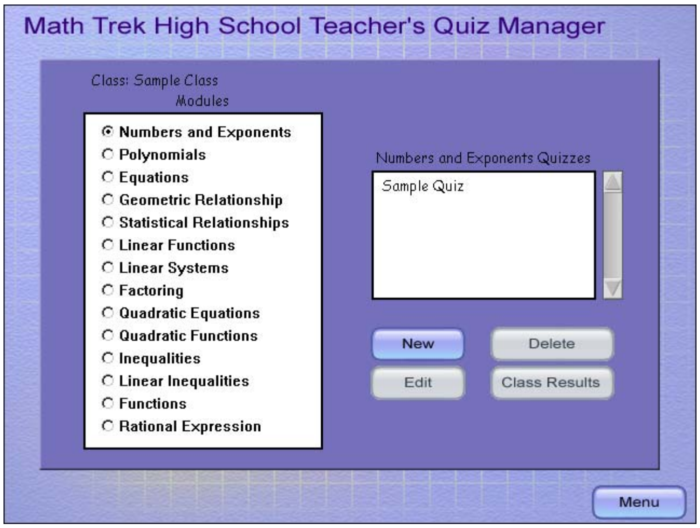
- Click on the **Teacher** button on the Login screen.
- Click the **Change** button.
- Enter the current password.
- Enter the new password in the space provided.
- Verify the new password (retype the password).
- Click the **OK** button.
- Record the password somewhere in writing in case it is forgotten.

Quizzes

Each section contains a Quiz topic. After selecting this option, the student has three choices available.

1. Practice: Students select the questions for practice. Each question is scored with a show answer and an Explain feature available for incorrect answers.

2. Sample Quiz: A set number of questions is provided; however, the values in the questions are randomized. Therefore, the sample quiz can be retaken. Student scores are tracked.
3. Teacher Quiz: The teacher must pre-select the questions and save a quiz before students can access a Teacher Quiz. The values in the questions are randomized. Student scores are tracked.



Teacher Quiz Screen

RESOURCE MANUAL

The full version of this program includes a comprehensive teacher resource manual. It consists of:

- software overview;
- scope and sequence of skills and knowledge by topic and section;
- additional resources for the classroom teachers including teaching strategies and blackline master templates, assessment instruments, performance tasks;
- strategies for extending the software throughout technology applications;
- glossary.

AVAILABLE LICENSES

The license which accompanies the software outlines how the program may be used, either on 1 computer, 10 computers, as a site license, or board/district license. Licenses may be upgraded at any time. Contact NECTAR for details.

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Approach to Mathematics

OVERVIEW

Students demonstrate diversity in the ways they best learn mathematics; therefore, they must have opportunities to learn in a variety of ways: individually, cooperatively, independently, directed by the teacher, through hands-on experience, and through examples followed by practice. Math Trek High School provides the flexibility to use a variety of learning techniques, including modelling of examples and extensive practice opportunities through randomly generated questions.

Various teaching strategies are essential for effective learning. However, all learning should be embedded in a context. Rich learning environments allow students to understand major underlying principles, thus encouraging the development and use of mathematical reasoning. Performance tasks in the Math Trek High School manual are included to compliment the software's focus on mathematical skills and knowledge by emphasizing higher order thinking skills. Variety in teaching strategies that address multiple intelligences and different learning styles are critical for mathematical learning. Software can provide audio support and interactive manipulations making mathematics accessible to auditory, visual, and tactile learners. Van DeWalle describes mathematics software this way: "The best of these programs allow students to freely manipulate the computer model with keystrokes or a joystick, thus simulating actual manipulation of physical materials." The flexible nature of software allows students to progress at their own rate and allows the teacher to identify specific skills for reinforcement or enrichment.

Student attitudes, interests, needs and feelings play a crucial role in mathematics teaching and learning. If teachers and students enjoy mathematics and have a positive attitude, learning will be facilitated. Software provides such motivation through animation, graphics, engaging activities, and audio support. Randomly generated examples and problems with immediate feedback and explanation engage the learner and promote success.

The curriculum must equip students with essential mathematical knowledge and skills, as well as the skills of reasoning, problem solving and communication, and the ability to use technology effectively to process quantitative information. Mathematical knowledge becomes meaningful and powerful in application. Software that combines a sound sequence of skills and knowledge with real world problem solving and communication of mathematics learning can effectively address the curriculum.

a) **Mathematics as problem solving**
Problem solving is a vehicle for learning mathematics. Tutorials, applications, and open-ended tasks give students opportunities to select their own approaches for both problem solving and expressing mathematical ideas.

b) **Mathematics as communication**
Many teachers have reported positive results from journal writing in which students reflect on their experiences in learning mathematics and a proof of true learning. On-line journals facilitate this type of communication.

c) **Mathematics as reasoning**
Logical, spatial, and geometric reasoning can be inherent to the tutorials and exploratory activities in software programs. The complexity of approach must reflect the development of students' reasoning abilities over time; the content must be grade and age appropriate.

d) **Mathematical connections**
The contexts for development of mathematical skills and knowledge in software can relate to other disciplines (e.g., language, social studies, business, physical education and the arts).

"Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (NCTM, 2000) Learning tools, such as a calculator extend the range of problems and allow students with special needs to focus on important procedures.

Multiple strategies can be used to integrate software with the Mathematics curriculum.

1. For whole class instruction on the delivery of a new, or expanded concept, the teacher can use software for tutorial and demonstration purposes. A single computer connected to a projection device, such as an LCD projector or television, provides a focus. The teacher has available, at the click of a mouse, unlimited access to numerous examples, clear graphs and precise diagrams. Both tool-based and curriculum-based software is appropriate for this strategy.
2. For individualized instruction, students can proceed at their own pace through tutorial portions of curriculum-based software. Programs that provide immediate feedback and reinforcement are effective for individualized or independent instruction. This flexible strategy is appropriate for teachers who may have limited access to a computer lab or who may have a small pod of computers in their classrooms.
3. Students requiring remediation or enrichment can have their individual needs addressed through the integration of software, either tool-based or curriculum-based. When using curriculum-based programs students are directed to specific sections to review the concepts and practice their skills or to extend their skills through challenging, more complex applications. This strategy does not require consistent access to a large bank of computers.
4. Software is an additional resource for teachers to use to assist students in developing their skills and knowledge. The use of software for practice is effective when the programs provide randomly-generated questions with immediate feedback and explanations.
5. Software is a useful technique to use to assess students' mathematical achievement. Math Trek High School provides a reporting feature that tracks scores as well as the specific skills and concepts providing difficulties for individual students. Thus, it assists the teacher to work with the student to set goals and improve learning.

Regardless of the frequency and extent of the access to computers, there are strategies that allow teachers to effectively integrate software and the Mathematics curriculum in order to improve students' skills, to deliver mathematical concepts, and to assess student achievement.

PROBLEM SOLVING

Problem solving is not considered a separate topic. Students must be engaged in problem-solving regardless of the skills and knowledge being addressed.

A positive attitude, strategies and processes combined with reflecting on one's own thinking should be characteristic of effective problem solving.

Students are able to realize the power and usefulness of mathematics through on-going problem solving activities connecting the real world to the academic world.

Problem solving is a trial-and-error process that involves starts and stops, successes and failures, and the examination and rejection of some solutions. There are problem solving strategies that can be taught. It is critical that students be required to explain their strategies, processes and solutions using correct mathematical language.

A suggested model is outlined below:

1. Understanding the problem.

   The student ....

   i. knows the meaning of the words in the problem;
   ii. identifies key words;
   iii. draws a diagram;
   iv. identifies insufficient, required, and extraneous information;
   v. rereads and restates the problem in own words;
   vi. uses concrete manipulatives ;

vii. looks for patterns;
viii. discusses the problem to gain a better understanding.

2. Developing a plan.
   The student ....
   i. compares the problem to previous experience;
   ii. considers possible strategies;
   iii. guesses, checks and improves the guess;
   iv. selects a strategy;
   v. discusses the strategy.

3. Carrying out the plan.
   The student ....
   i. executes a chosen strategy;
   ii. presents ideas clearly;
   iii. does the calculations;
   iv. monitors success;
   v. revises if necessary.

4. Looking back.
   The student ....
   i. determines if the answer is reasonable;
   ii. explains the answer in oral and written form;
   iii. states the solution to the problem;
   iv. restates the problem with the answer;
   v. considers alternate solutions.

THE JOURNAL

In *Math Trek High School* the journal is accessed by clicking the TOOLS button and selecting journal.

The journal is an important part of the *Math Trek High School* software program. Through the journal the teacher, the student and parent can gain insights into the student's progress and attitude.

The journal helps the teacher in many ways. It can be used for teacher-student conferences. Through these discussions, the teacher gains an understanding of the student's progress. In addition, the journal is an excellent vehicle for the student to show an understanding of the process that was used to arrive at a solution. Journal entries can also convey the alternate ways that students use to solve problems and provide an opportunity for them to reflect on their learning styles. By analyzing and reflecting on their progress, the students gain confidence in their mathematics abilities.

The journal in *Math Trek High School* can be used in a number of different ways. It can be used:

- to express feelings about mathematics;
- to provide a vehicle for questions;
- to reflect on problem solving strategies;
- to identify real-world applications of mathematics;
- to list alternate ways of finding solutions;
- to answer specific questions/problems based on expectations/skills.

Introducing The Journal

For many of the students the journal may be a new experience and it will be important to introduce it and establish expectations or guidelines for its use.

1. The first step is to discuss with the students the reasons for maintaining a journal. The various benefits associated with the journal should be explained so that students can embrace this learning tool.

2. The second step is to establish guidelines and expectations with the students.

3. The third step is to reassure the students that you will respect their feelings and concerns and that you will read their entries. You will need to consider how you will respond to their journals. It is not necessary for the students to make daily entries but do plan to set up a plan so that all students have a regular chance to work with the journal.

4. The fourth step is to set up a format for students to follow. Example: Will you want the entries dated? Will you be posing questions for the students to answer? etc.

5. Assign specific journal entries. These can vary from explaining solutions and discussing strategies to posing questions and analyzing processes.

The journal is an integral part of the software package and can be used to enhance the students' abilities to use mathematical knowledge and skills, think critically, and solve problems. Each section includes journal activities.

MULTIPLE INTELLIGENCES

It is important that educators are cognizant of the Multiple Intelligences and how the students' experiences tap into all areas. Howard Gardiner, a Harvard University psychologist, believes that all individuals have seven or more distinct intelligences. One or more of the intelligences may be stronger than the other.

Spotting the Seven Intelligences

Linguistic Intelligence involves ease in producing language and sensitivity to the nuances, order and rhythm of words. Students who exhibit linguistic intelligence love to read books, write and tell stories. They have good memories for names, places, dates and trivia.

Logical-Mathematics Intelligence relates to the ability to reason deductively or inductively and to recognize and manipulate abstract patterns and relationships. Students who excel at math, have strong problem-solving and reasoning skills and ask questions in a logical manner exhibit this intelligence.

Spatial Intelligence is the ability to create visual-spatial representations of the world and to transfer those representations mentally and concretely. Students who exhibit spatial intelligence need a mental or physical picture to best understand new information; do well with maps, charts, and diagrams; and like mazes and puzzles. They can design, draw, and create things.

Musical Intelligence includes sensitivity to the pitch, timbre and rhythm of sounds, and responsiveness to the emotional implications of these elements. Students who remember melodies or notice pitch and rhythm exhibit musical intelligence. They tend to be aware of surrounding sounds.

Bodily-Kinaesthetic Intelligence involves using the body to solve problems, create products, and conveys ideas and emotions. Students who exhibit bodily-kinaesthetic intelligence are good at physical activities and have a tendency to move around, touch things, and gesture.

Interpersonal Intelligence refers to the ability to work effectively with other people; to understand them; and to notice their goals, motivations, and intentions. Students who thrive on co-operative work, have strong leadership skills, and are skilled at organizing, communicating, meditating, and negotiating exhibit this intelligence.

Intrapersonal Intelligence entails the ability to understand one's own emotions, goals, and intentions. Students who exhibit this intelligence have a strong sense of self, are confident, and often prefer working alone. They also have good instincts about their strengths and abilities.
Taken from The Instructor, July/August, 1992, pp 48-49.

No one school program can do justice to the different intelligences but it is important to look at the different classroom strategies and assessments so that a balanced program can be put in place. In the traditional school environment the emphasis is on the linguistic and logical-mathematical intelligences. In *Math Trek High School* a number of the intelligences are addressed.

Students recognize the true relevance of their learning when they see real world connections to their work.

Verbal- Intelligence - Example: In *Math Trek High School* the students can communicate in writing using the journal. The exercise may be in response to a question posed by the teacher or initiated by the students.

Logical-Mathematical Intelligence - Example: In *Math Trek High School* there are many opportunities to develop high-order reasoning particularly in the activities component of the software. Mental math skills are integrated into the learning tasks and of course numerous calculation tasks.

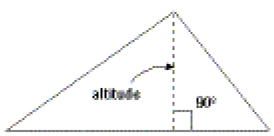
Visual-Spatial Intelligence - Example: In *Math Trek High School* the students work on reading and understanding graphs and diagrams.

Interpersonal Intelligence - Example: With *Math Trek High School* the teacher can easily incorporate the opportunity to explain and teach another student a particular skill.

Intrapersonal Intelligence - Example: Thoughts and feelings and personal goals re: mathematics can be included in reflections in the journal. *Math Trek High School* includes reflective questions.

MATHEMATICS GLOSSARY

acute angle
An acute angle is an angle whose measure is greater than 0º and less than 90º.
acute triangle
An acute triangle is a triangle with three acute angles.
additive inverse
The additive inverse of a number n is *–n; i.e. n + (-n) = 0*
algebra
Algebra is the study of the general properties of numbers.
algebraic equation
An algebraic equation is an equation that involves symbols and numbers.
algebraic expression
An algebraic expression is a collection of symbols representing numbers and operations.
alternate interior angles
Alternate interior angles are pairs of angles formed on opposite sides of a transversal when it crosses two lines. Alternate interior angles are located between the two lines crossed. Alternate interior angles do not share a vertex.
altitude
An altitude of a triangle is a line segment that connects a vertex to the opposite side and is perpendicular to the opposite side. The altitude represents the height of the triangle. For example:



angle
An angle is the union of two rays that have a common endpoint. The rays are called the sides of the angle and the common endpoint is called the vertex of the angle.
angle bisector
An angle bisector is a ray that divides an angle into two congruent angles.
aperiodic tiling
Aperiodic tiling is a combination of two or more tiles that tile a plane with a pattern that does not repeat.
area
Area is the amount of surface of an object. When measured in square units, the area is the number of squares needed to completely cover the surface. Formula: **area of a rectangle:** A = *lw*; where *l* is the length, and *w* is the base of the rectangle **area of a triangle:** A = $\frac{bh}{2}$; where *b* is the base, and *h* is the height of the triangle.
associative property
An operation such as addition or multiplication for a set of numbers is associative if changing the grouping of the numbers being operated upon does not change the result. Example: Addition of integers is associative, since for any 3 integers a, b, c (a + b) + c = a + (b + c).
arithmetic sequence
An arithmetic sequence is an ordered list of numbers, in which the difference between any term in the sequence and the term before it is constant.
attribute
An attribute is a quantitative or qualitative characteristic of an object or a shape; for example, colour, size, thickness.
average
Same as mean. The mean of a set of numbers is the sum of the numbers divided by however many numbers there are.

bar graph

A bar graph is a type of graph that represents a number of data elements grouped into categories.

base (of a power)

In algebra, the base is the number in a power that represents the number being repeatedly multiplied.

BEDMAS

BEDMAS is an acronym for the order of doing operations – Brackets, Exponents, Divide/Multiply, Add/Subtract.

bias

A bias is any influence on data collection that might result in a sample being unrepresentative of the population from which the sample is taken.

binomial

A binomial is a sum or difference of two monomials. Example: 4xy + 2.

bisector

A bisector is anything that divides a line segment, an angle, or a figure into two congruent halves.

box-and-whisker plot

A box-and-whisker plot is a chart used to display the distribution of data in horizontal rows.

capacity

Capacity is the volume of a container usually measured in cubic centimetres or litres.

census

A census is a survey of the complete population as opposed to a sample survey (which will collect data from a portion of the population).

centroid of a triangle

The centroid of a triangle is the point where its three medians intersect.

chevron

A chevron is a quadrilateral whose diagonals do not intersect.

circle

A circle is a set of all the points in a plane that are equidistant from a given point in the plane (the center of the circle).

circular cone

A circular cone is a cone whose base is circular.

circle graph

A circle graph is a type of graph that represents a number of data elements grouped into categories. Each category is represented by a sector of the circle. The size of each sector is proportional to the percent of all of the data that is part of the category the sector represents.

circumference

A circumference is the boundary (perimetre) of a two-dimensional geometric shape.

closure property

A set is closed if an operation on any element in the set results in another element that is in the set.

coefficient

A coefficient is the numerical factor of a term. For example, in the term $5x$, the coefficient is 5; in the term ax, the coefficient is a.

common denominator

A common denominator is a common multiple of the denominators of two or more fractions.

commutative property

An operation such as addition or multiplication for a set of numbers is commutative if changing the order of the numbers being operated upon does not change the result. Example: Addition of integers is commutative, since for any 2 integers a and b, a + b = b + a.

complementary angles

Complementary angles are two angles whose sum is 90 degrees. The angles do not necessarily have to share a common ray.

complementary event
An event is complementary to another if the two events taken together represent all possible outcomes.
complex event
A complex event is an event (outcome) consisting of 2 or more simple events. Example: The outcome of flipping two coins is a complex event.
composite numbers
Composite numbers are whole numbers that have more than two factors.
cone
A cone is a three dimensional figure formed by a circular base and all the line segments connecting that base to a fixed point of vertex not on the base.
congruent
Congruent means to have the same measure and shape.
congruent figures
Congruent figures are figures that have the same shape and size.
continuous data
Continuous data is data whose values can be expressed to any degree of precision.
Example: height, temperature, distance, time.
coordinates
Coordinates are a set of numbers that define a position relative to a set of axes or some other frame of reference.
coordinate plane
A coordinate plane is a grid where coordinates can be plotted.
correlation
A correlation is a description of the general relationship between two quantities. A correlation can be positive (when one quantity increases, the other increases) or negative (when one quantity increase, the other decreases), strong, weak, or non-existent.
corresponding angles
For lines m and n cut by transversal t, two angles which are on the same side of the transversal are called corresponding angles if one is an exterior angle, one is an interior angle, and they have different vertices.
co-interior angles
For line m and n cut by transversal t, two angles are called co-interior angles if they are on the same side of the transversal and are both interior.
cube
A cube is a six sided Platonic Solid. Its faces are all squares. A cube is also a hexahedron.
cylinder
A cylinder is a three dimensional figure having two congruent, parallel and circular bases joined by a curved lateral surface.
data
Data is facts information, or concepts for the purpose of drawing conclusions.
database
A database is an organized and sorted list of facts or information; usually manipulated by a computer.
decimal
A decimal is a base ten number, generally used for numbers with a fractional part, indicated by a decimal point.
deductive reasoning
Deductive reasoning is the process of reaching a specific conclusion based on general facts.
denominator
A denominator is the bottom number or divisor of a fraction. Example: The denominator of the fraction ½ is 2.

dependent event
A dependent event is an event whose probability of occurrence is influenced by another. Example: if out of a bag of red and green candies, a red candy is drawn and not put back, this changes the probability of drawing a green candy.

diametre
A diametre is a line segment that joins two points on the circumference of a circle and passes through the center of the circle.

difference
A difference is the result of subtracting one number from another.

dilatation
A dilatation is a type of transformation that maintains the same shape but may result in a change in size. Enlargements and reductions are examples of dilatations.

direct variation
A direct variation describes a relationship between two variables in which one variation is a constant multiple of the other.

discrete data
Discrete data is data that can only have certain values. Example: the possible values of the number of pets a person owns can only be a whole number.

discriminant
A discrimination is the part of a quadratic formula that indicates the number of solutions to a quadratic equation.

distributive property
The distributive property is a property of numbers wherein a product of a sum can be written as a sum of the products.

dividend
In a division problem, the dividend is divided by the divisor.

divisor
In a division problem, the dividend is divided by the divisor.

dodecahedron
A dodecahedron is a twelve sided Platonic Solid. Its surfaces are all pentagons.

domain
The domain of a relation is a set of the first members of a set of ordered pairs. Example: {1, 3, 6} is the domain of {(1, 2), (1, 6), (3, -4), (6, 6), (6, 0)}

elimination
Elimination is an algebraic method of solving a linear system in which one variable is "eliminated" from the system by adding or subtracting the multiples of the equations.

equation
An equation is a mathematical statement that has equivalent expressions on either side of the equal sign. Example: $y = 3x + 2$.

equilateral triangle
An equilateral triangle is a triangle with three congruent sides. An equilateral triangle is also an isosceles triangle.

equivalent
Equivalent means to have equal value.

equivalent fractions
Equivalent fractions are fractions that are equal in value but have different denominators.

equivalent rate
An equivalent rate is a rate that is equivalent to another.

evaluate
To evaluate means to compute a value. To evaluate an algebraic expression, values are substituted for the variables and the computations are performed.

expanded form
Expanded form is a way of writing numbers that shows the value of each digit. Example: The number 546 is shown in standard form on the left hand side of the equal sign and in expanded forms on the right hand side of the equal sign, with base 10.

$546 = 500 + 40 + 6$

$= 5 \times 10^2 + 4 \times 10^1 + 6 \times 10^0$
$= 5 \times 100 + 4 \times 10 + 6 \times 1$

experimental probability
Experimental probability is a measure of the relative frequency of the occurrence of an event derived from observations.

exponent
An exponent is a number in a power used to indicate how often the base is used as a factor. For example, 5^4 means 5 x 5 x 5 x 5.

exponential form
Exponential form is the form of a number when expressed as a power. Example: An exponential form of 1000 is 10^3.

exterior angle
For lines m and n cut by transversal, the exterior angles are the ones not between the lines m and n.

extrapolation
Extrapolation is the process of estimating a value lying outside the range of known values.

factor
A factor is any one of the numbers used in multiplication to form a product.

factored form
A number or expression is in factored form when it is written as a product.

figure
A figure is a set of points. A plane figure is a set of points in which all points are in the same plane.

formula
A formula is a set of ideas, words, symbols, figures, characters, or principles used to state a general rule. Example: The formula for the area of a rectangle is A = l x w.

fraction
A fraction is a ratio of two numbers where the second number is not zero; for example, ¾ is a fraction.

frequency
Frequency is the number of times an event or item occurs.

function
A function is a set of ordered pairs with no two elements the same.

general form
General form refers to linear equations in the form Ax + By + C = 0 in which A, B, and C are integers.

graph
A graph is a representation of data in a pictorial form. Some types of graphs are: bar graph, broken-line graph, circle graph, and pictograph.

greatest common factor (GCF)
The greatest common factor is the largest number by which two or more numbers can be divided without remainders. Example: The GCF of 24 and 30 is 6.

histogram
A histogram is a chart used to display grouped data in which the width and area of the bars are part of the data analysis.

horizontal axis
A horizontal axis is a horizontal line on a coordinate plane passing through y = 0. It is often called the x – axis.

horizontal coordinate
A horizontal coordinate is the first number of an ordered pair giving the position in Cartesian coordinates.

horizontal line
A horizontal line is a line parallel to the x – axis, whose slope is 0 (i.e., it has no rise). It is represented as an equation in the form y = c, (c is a constant).

icosahedron
An icosahedron is a twenty sided Platonic Solid. Its faces are all triangles.

improper fraction

An improper fraction is a fraction in which the numerator is greater than the denominator.

independent event

An independent an is an event whose probability of occurrence is not influenced by another. Example: If a die is tossed and a coin is flipped, the event of getting heads on the coin is independent of the outcome of rolling the die.

incenter

The incenter of a triangle is the point where its three angle bisectors intersect.

incircle

An incircle is a circle drawn inside a triangle whose center is the incenter of the triangle. The circle touches each side of the triangle just once.

inductive reasoning

Inductive reasoning is the process of reaching a general conclusion based on specific facts.

inequality

An inequality is a number statement or algebraic statement which contains one of these signs: not equal to, less than, greater than, less than or equal to, greater than or equal to.

infinite

Infinite refers to having no limit. Example: The set of whole numbers is infinitely large because no matter how large a number is, a larger number can be obtained by adding 1 to it.

integers

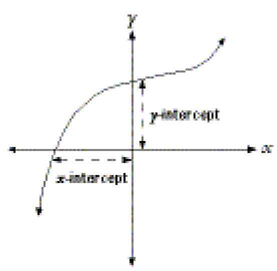
Integers are the set of positive and negative natural numbers and zero.

integral value

An integral value is a value which is an integer.

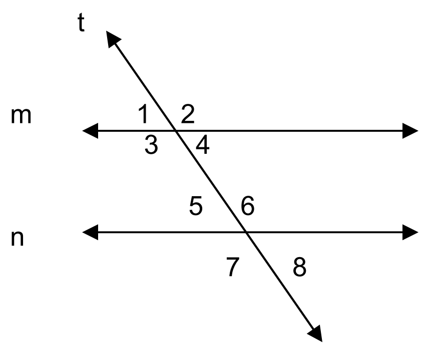
intercepts

An x-intercept is the x-coordinate of a point where a graph intersects the x-axis. A y-intercept is the y-coordinate of a point where a graph intersects the y-axis. For example:



interior angle

For lines m and n cut by transversal t, as in the diagram below, angles 3, 4, 5 and 6 are interior angles.



interpolation
Interpolation is the process of estimating a value between two known values.
intersecting lines
Intersecting lines are two lines with exactly one point in common, the point of intersection.

intersection point
An intersection point is a point that is common to two or more geometric figures.
inverse operations
Inverse operations are opposite operations (e.g. subtraction is the inverse operation to addition).
irrational numbers
Irrational numbers are numbers whose decimal form is non-terminating and non-repeating. Irrational numbers cannot be expressed as a quotient of two integers. The set of irrational numbers combined with the set of rational numbers forms another set called real numbers. Example: The non-terminating and non-repeating decimal number π is an irrational number.
irregular polygon
An irregular polygon is a polygon whose sides and angles are not all congruent.
isolate
In solving an equation, arithmetic operations are used to get the variable term only on one side of the equation.
isosceles triangle
An isosceles triangle is a triangle with at least two congruent sides. All equilateral triangles are also isosceles triangles.
kite
A kite is a quadrilateral with two pairs of congruent and adjacent line segments.

least common multiple
The least common multiple is the smallest common multiple of two numbers.
light year
A light year is a unit of length for measuring astronomical distances; one light year is the distance light travels in one year – approximately 9 460 528 404 846 km
like terms
Like terms have exactly the same variable expression. e.g. $2x$ and $3x$ are like terms. $2x$ and x^2 squared are NOT like terms.
line of symmetry
A line of symmetry is a line which divides a plane figure into two congruent parts.
line segment
A line segment is a part of a line joining two endpoints.
linear
Linear means in a straight line.
linear dimension
A linear dimension is a dimension involving the measurement of only one linear attribute, such as length, width, height, or depth.
linear equation
A linear equation is an equation of the first degree. Linear equations have straight line graphs. Example: y = 2x + 3
linear relation
A linear relation is a relation which, when plotted on a Cartesian plane, all of whose points lie on a straight line.
linear system
A linear system is a set of at least two linear (first degree) equations.
line
A line is a set of points in a straight path extending infinitely in two opposite directions along the path.

line of best fit
A line of best fit is a line drawn through a set of data displayed on a scatter plot that best fits or summarizes the information.

literal coefficient
A literal coefficient is a letter used as a coefficient in an algebraic term.
E.g. If x is a variable, the literal coefficient in the term mx is m.

lowest common denominator (LCD)
The lowest common denominator is the smallest number which is a multiple of all the denominators under consideration.

lowest terms
When a fraction is in lowest terms, the numerator and the denominator have no common factor other than 1.

mathematical modelling
Mathematical modelling is a mathematical description or model of a situation that may be a graph, equation, or algebraic function expressing the relationship between variables.

mean
Mean means average. The mean of a set of numbers is the sum of all the numbers divided by however many numbers there are. Example: 6 + 7 + 7 + 8 = 28. 28 ÷ 4 = 7. 7 is the mean.

measure of central tendency
Measures of central tendency are measures that attempt to represent the central trend of a data set. Mean (average), median (middle value), and mode (most common value) are all measures of central tendency.

median
In data management, the median is the middle value in a set of data that has been ranked from lowest to highest. In geometry, a median of a triangle is a line segment that connects a vertex to the midpoint of the opposite side.

midsegment
A midsegment is a line segment that joins the midpoints of two adjacent sides of a polygon.

mixed number
A mixed number is a number that is the sum of an integer and a proper fraction.

mode
The mode is the value that occurs most often in a data set.

monomial
A monomial is the product of a number and variable(s), in which the exponents on the variable(s) are whole numbers. Examples: 3x, 29ab, 6.

multiple
A multiple is any product of a number and a natural number. Example: 4, 8, 12 … are multiples of 4.

multiplication
Multiplication is an operation that combines numbers called factors to give one number called a product. Example: 4 x 5 = 20; factor x factor = product.

natural numbers
Natural numbers are the numbers 1, 2, 3 … also called the counting numbers. This set is usually represented by the letter N.

negative numbers
Negative numbers are the set of numbers less than zero.

net
A net is a two-dimensional shape that can be folded to make a three-dimensional figure.

non-standard units
Non-standard units are units of measurement other that standard units of measure. Example: hand spans (non-standard) and metres (standard).

numerical expression
A numerical expression is a number statement consisting of some combination of numbers and operations

number line

A number line is a line on which all the real numbers are represented by points. All numbers are represented by points according to their distance from, and direction left or right of, a point chosen as zero.

number sentence

A number sentence is a statement comprising numbers, operations, and an equal sign. Example: 12 – 8 = 4

numerator

A numerator is the top number or dividend in a fraction. In a fraction of the form $\frac{a}{b}$, a is called the numerator.

numerical coefficient

A numerical coefficient is the constant (non-variable) factor of an algebraic term. Example: In 3x, the numerical coefficient is 3.

oblique cone

An oblique cone is a circular cone whose vertex does not lie directly over the center of the base.

oblique cylinder

An oblique cylinder is a cylinder whose curved face is not perpendicular to the bases.

obtuse angle

An obtuse angle is an angle whose measure is greater than 90° and less than 180°.

obtuse triangle

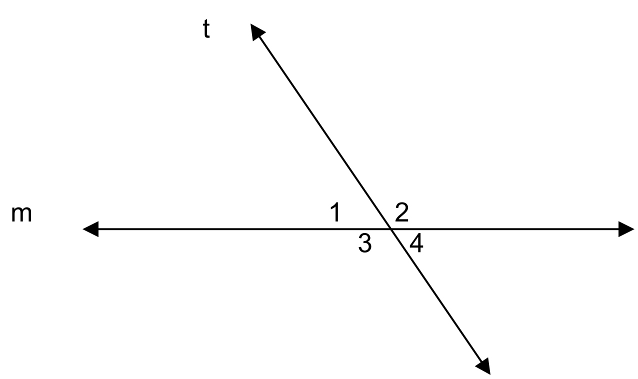
An obtuse triangle is a triangle with an obtuse angle.

octahedron

An octahedron is an eight sided Platonic Solid. Its faces are all triangles.

opposite angles

Opposite angles are pairs of congruent angles formed by two intersecting lines. Example: In this diagram angles 1 and 4 are opposite angles. Angles 2 and 3 are also opposite angles.



order of operations

The order of operations is the set of rules regarding the order in which arithmetic operations must be performed. See also: BEDMAS.

ordered pair

An ordered pair is a pair of numbers in which the first number represents a point on the horizontal axis (x-axis) and the second number represents a point on the vertical axis (y-axis) on a coordinate plane. See also: coordinates. Example: (5, -2).

ordinal number

An ordinal number is a number that shows relative position or place. Example: first, second, third, fourth.

origin

The origin is the point (0, 0) on a coordinate plane where the horizontal axis (x-axis) and the vertical axis (y-axis) meet.

orthocenter

The orthocenter of a triangle is the point where its three altitudes intersect.

parallel lines
Parallel lines are lines in the same plane, which do not intersect.

parallelogram
A parallelogram is a quadrilateral whose opposite sides are parallel. The opposite sides of a parallelogram are congruent. The opposite angles of a parallelogram are also congruent.

partial variation
A partial variation describes a relationship between two variables in which one variable is a constant multiple of the other plus another constant.
Example: $y = 3x + 4$

percent
Percent is the number of parts per 100. The symbol for percent is %.

perfect square
A perfect square is the result of a number multiplied by itself. Example: 16 is a perfect square, since it is the product of 4 and 4.

periodic tiling
Periodic tiling is a combination of two or more tiles that form a pattern that tiles the plane.

perpendicular lines
Perpendicular lines are two lines that intersect at a 90 degree angle.

perimetre
A perimetre is the boundary of a two-dimensional geometric shape. For a circle, perimetre and circumference have the same meaning.

pi
Pi is the ratio of a circle's circumference to its diametre. It is a non-terminating, non-repeating decimal approximately equal to 3.14. Pi is an irrational number. The symbol for pi is π.

pictograph
A pictograph is a type of graph that represents a number of data elements grouped into categories. Within each category, one or more pictures are used to represent the number of elements in the category.

place value
Place value is the value of the position of a digit within a number. In the decimal number system the place values are ones, tens, hundreds, thousands, or tenths, hundredths, etc.

platonic solid
A Platonic Solid or regular solid is a polyhedron all of whose faces are congruent regular polygons, where the same number of faces meet at every vertex. There are only five Platonic Solids: tetrahedron, cube, octahedron, dodecahedron, icosahedron.

plane
A plane is a flat surface that extends infinitely in all directions.

point
A point is a geometric object that specifies a location in space. It has position but no size. A point is usually represented by a dot.

polygon
A polygon is a closed two-dimensional figure formed by three or more line segments.

polyhedron
A polyhedron is a three-dimensional figure whose faces are all polygons.

polynomial
A polynomial is an algebraic expression consisting of a monomial or sum or difference of monomials.
Example: $3x$, $a + b$, $4h^2 - 2k^3m$

population
A population is the complete set of items being studied.

positive numbers
Positive numbers are the set of numbers greater than zero.

power
A power is a product of identical factors. A power is usually expressed use a base and an exponent; for example, 3^4 is a power.

prime number
A prime number is a whole number greater than 1 that has only two factors, itself and 1.

prism
A prism is a polyhedron with at least one pair of congruent and parallel faces (bases).

probability
Probability is a number between 0 and 1 inclusive, that indicates the likelihood of an event occurring. A probability of 0 means that there is no possibility of the event occurring. A probability of 1 means that the event is certain to occur.

product
A product is the result of multiplying two or more numbers (factors). Example: 5 x 7 = 35, 35 is the product.

proper fraction
A proper fraction is a fraction in which the numerator is smaller than the denominator.

proportion
A proportion is an equality between two equivalent ratios.

protractor
A protractor is a tool used to measure angles.

pyramid
A pyramid is a polyhedron formed by a polygonal base and a number of lateral triangular faces that meet at a common vertex.

pythagorean theorem
The Pythagorean Theorem describes a relationship that exists between the lengths of the three sides of any right triangle: the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

quadrant
A quadrant is any one of the four parts of a coordinate plane formed when the horizontal axis (x-axis) and the vertical axis (y-axis) cross.

quadratic equation
A quadratic equation is an equation that can be rewritten in the form $ax^2 + bx + c = 0$ where a, b, and c are constants and a $\neq 0$.

quadratic functions
Quadratic functions are functions that can be written in the form of $y = ax^2 + bx + c$ where a, b, and c are constants and a is not 0.

quadrilateral
A quadrilateral is a four sided polygon.

qualitative variables
Variables that cannot be measured numerically (e.g. hair colour, taste of food)

quantitative variables
Variables that can be measured numerically (e.g. weight, shoe size, time)

quotient
A quotient is the result of dividing one number by another. e.g. $20 \div 4 = 5$, 5 is the quotient.

radius
A radius is a line segment whose endpoints are the center of a circle and a point on the circle's circumference.

random
Random means a selection in which each possible outcome has the same probability. Example: The selection of one person from a class of students is random if every person has the same probability of being selected.

random sample
A random sample is a sample in which every member of the population has an equal chance of being chosen.

range
The range of a relationship is the set of the second members of a set of ordered pairs. Example: {-4, 0, 2, 6} is the range of {(1, 2), (1, 6), (3, -4), (6, 6), (6, 0)}. In data management, range is the difference between the highest and lowest numbers in a group of numbers. In the data set {8, 32, 25, 20}, the range is 24; that is 32 - 8.

rate

A rate is a ratio of two quantities having different units. Example: The ratio of distance to time is a rate, called speed. Speed could be expressed in kilometres per hour.

ratio

A ratio is a comparison of two or more numbers; as in a:b, or a/b. Example: 4:1 is a comparison of the number 4 to the number 1.

rational number

A rational number is any number that can be expressed as the quotient of two integers where the second number is not zero. Q is used to represent this set of numbers.

rational expression

An expression that can be written as a quotient of two polynomials; for example, if P and Q are two polynomials, and $Q \neq 0$, then $\frac{P}{Q}$ is called a rational expression.

ray

A ray is part of a line extending infinitely from a given point in one direction only.

real numbers

Real numbers are the union of rational and irrational number sets. Real numbers include all positive and negative integers, zero, fractions with numerators and denominators that are integers, and decimal numbers.

reciprocals

Reciprocals are pairs of numbers whose product is 1. E.g. $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$.

rectangle

A rectangle is a quadrilateral with four right angles. Each pair of opposite sides of a rectangle is congruent and parallel.

rectangular prism

A rectangular prism is a polyhedron with at least two congruent, parallel, and rectangular faces (bases).

reflection

A reflection is a transformation that reflects a figure over a mirror line. The figure does not change size or shape, but it does change position and orientation. Also called a flip.

reflectional symmetry

A figure has reflectional symmetry if it can be folded along a line so that it folds onto itself.

reflex angle

A reflex angle is an angle whose measure is greater than 180 degrees and less than 360 degrees.

regular polygon

A regular polygon is a polygon in which all sides are congruent and angles are congruent.

relation

A relation is a set of ordered pairs. See also: function. Example: {(1, 2), (1, 6), (3, -4), (6, 6), (6, 0)}.

relatively prime

When two numbers have no common factors other than 1, they are described as relatively prime. Example: The numbers 9 and 20 are relatively prime.

repeating decimals

Repeating decimals are decimal numbers that contain an infinitely repeating non-zero digit or block of digits. Examples: 0.11111, 2.121212.

representative sample

A representative sample is a sample that is representative of the population from which the sample is taken.

rhombus

A rhombus is a quadrilateral with four congruent sides. All rhombi are parallelograms.

right angle

A right angle is an angle whose measure is 90°.

right circular cone
A right circular cone is a circular cone whose vertex lies directly over the center of the base.
right cylinder
A right cylinder is a cylinder whose curved face is perpendicular to its bases.
right triangle
A right triangle is a triangle containing a right angle.
rotation
A rotation is a transformation that rotates or turns a figure about a fixed point. The figure does not change size, shape, or orientation. Also called a turn.
rotational symmetry
A figure possesses rotational symmetry if it can be rotated about its center so that it fits onto itself in less than a complete rotation.
scalene triangle
A scalene triangle is a triangle with three sides of different lengths. No two sides are congruent.
scatter plot
A scatter plot is a two-dimensional Cartesian graph displaying a set of paired observations.
scientific notation
A number in scientific notation is written as a product of two factors. One factor is a number from 1 to 10, including 1 but not 10, and the other factor is a power of 10; for example, 35000 is written as 3.5×10^4
sequence
A sequence is an ordered list of numbers with items in the list separated by commas.
SI
SI is an abbreviation for the international system of measurement units known as the metric system.
simplify
To simplify means to make less complex. E.g. $3x+5+4x-7=7x-2$
skeleton
A skeleton is a three-dimensional figure showing only the edges and vertices of the figure.
similar figures
Similar figures are figures that have the same shape but may not be the same size.
slope
Slope is a measure of the steepness of a straight line graph. Slope is expressed as a ratio of rise (vertical change) to run (horizontal change).
slope-intercept form
Slope-intercept form refers to linear equations in the form $y = mx + b$. The coefficients m and b represent the slope and y-intercept respectively.
solve
To solve an equation means to determine the values of the variables for which the equation is true.
sphere
A sphere is a closed curved surface, all points of which are equidistant from a fixed point at the center of the sphere.
square
A square is a quadrilateral with four right angles and four congruent sides.
square root
A square root of a number is another number, which multiplied by itself, equals the original number. Example: 4 is a square root of 16, since 4 x 4 = 16. -4 is also a square root of 16.
stem-and-leaf plot
A stem-and-leaf plot is a chart used to display the distribution of data in horizontal rows.
standard form
For numbers, standard form refers to the usual way a number is expressed, as opposed to expanded form. Example: 546.
straight angle
A straight angle is an angle whose measure is 180 degrees.

substitution

Substitution is an algebraic method of solving a system of equations in which a variable is isolated from one equation and its value substituted into another equation.

sum

A sum is the result of adding two or more numbers.

supplementary angles

Supplementary angles are two angles whose sum is 180°. The angles do not necessarily have to share a common ray.

surface area

Surface area is the measure of the number of square units required to cover all the surfaces of a three-dimensional object.

survey

A survey is a sampling of data collected from observations or from asking people questions.

symbols

+	add, plus	≥	greater than or equal to
-	subtract, minus	N	the set of natural numbers
x	multiply, times	W	the set of whole numbers
÷	divide	I	the set of integers
>	greater than	R	the set of real numbers
<	less than	Q	set of rational numbers
=	equal to	Q	set of irrational numbers
%	percent	≅	means is congruent to
≈	approximately equal to	~	means is similar to
≠	not equal to	°	degrees
≤	less than or equal to	{	used to indicate system of equations

system of equations

A system of equations is a set of equations.

table

A table is an orderly arrangement of facts set out for easy reference. Example: An arrangement of numerical values into rows and columns.

tally chart

A tally chart is a table used to record and count data.

term

A term is a part of an algebraic expression, which is separated from the rest of the expression by addition or subtraction signs. If a subtraction sign appears before a term, it is considered to be a negative term. Example: The algebraic expression 2x + 5xy - 3y has three terms. The terms are 2x, 5xy and -3y.

terminating decimals

Terminating decimals are decimal numbers that eventually end (with a repeating 0). Examples: 0.5, 3.765.

tessellation

Tessellation refers to the tiling of a plane.

tetrahedron

A tetrahedron is a four sided Platonic Solid. Its faces are all triangles.

theoretical probability

Theoretical probability is a predicted measure of the relative frequency of the occurrence of an event based on what should happen.

tiling a plane

Tiling a plane is the covering of a plane with shapes so that there are no overlaps or gaps.

translation

A translation or slide is a transformation in which each point of a figure moves the same distance in the same direction. The figure does not change size, shape, or orientation.

transformation
A transformation is a change in a figure that results in a different position, orientation, or size. Some common transformations are translations, rotations, reflections, dilatations, and sheers.
transversal
A transversal is a line (or line segment) that intersects two or more other lines (or line segments) in different points.
trapezoid
A trapezoid is a quadrilateral with at least one pair of parallel sides. All parallelograms are also trapezoids.
tree diagrams
Tree diagrams are diagrams that show the probability of a sequence of events occurring. Each segment of a tree diagram is assigned a probability.
triangle
A triangle is a polygon with exactly three sides.
triangular prism
A triangular prism is a three-dimensional shape composed of two triangular faces that are congruent and parallel. The triangles are joined together by rectangles.
trinomial
A trinomial is a three term polynomial.
unit rate
A unit rate is a rate in which the second term is 1.
variable
A variable is a quantity, usually represented by a symbol, that can take on any one of a set of values. Example: In the expression 3ab + c; a, b, and c are variables.
variable term
A variable term is a term involving a variable. Example: 2x.
vertex
A vertex is a point that at least two rays share as their starting point to form an angle.
vertical axis
The vertical axis is the vertical line on a coordinate plane passing through x = 0. It is often called the y-axis.
vertical coordinate
A vertical coordinate is the second number of an ordered pair giving the position of a point in Cartesian coordinates.
vertical line
A vertical line is a line parallel to the y-axis, whose slope is undefined (i.e., it has no run). It is represented as an equation in the form x = c, (c is a constant).
volume
Volume is a measure of the amount of space occupied by a three-dimensional object; measured in cubic units such as cubic centimetres or in litres.
whole numbers
Whole numbers are the set of numbers that consist of the set of natural numbers and the number zero.
***x*-intercept**
An x-intercept is the x-coordinate of a point where a graph intersects the x-axis.

***y*-intercept**
A y-intercept is the y-coordinate of a point where a graph intersects the y-axis.

TECHNOLOGY APPLICATIONS IN *MATH TREK HIGH SCHOOL*

*Technology is essential in teaching and learning mathematics;
it influences the mathematics that is taught and enhances students' learning.*

Technology helps to give students the means to explore mathematical concepts more easily and quickly. Students can study fundamental ideas in greater depth, and can concentrate their effort in the areas of data collection, data analysis, simulations, and complex problem solving.

- In *Math Trek High School,* an on-screen calculator is available at the touch of a button, when the student requires it to assist in calculation. Time is used more proficiently to understand concepts instead of wasting time crunching numbers.

- *Math Trek High School* provides dynamic exploratory environments to enhance students' ability to understand patterns, analyze change, apply transformations, display and analyze data, and solve problems.

- Animation and graphics are used extensively in all modules in *Math Trek High School* to assist students to move from a concrete understanding of mathematical concepts to the abstract and from specific examples to general ideas and formulae.

- *Math Trek High School* provides instant feedback, immediate explanations, and an unlimited supply of randomly generated questions. When students are working in exercises or tutorials they are presented with a Replay, Done, Mark, Show Answer, and Explain button. Using the Replay button students can produce an endless variety of similar questions for drill and practice. The Done or Mark button gives the student instant reinforcement regarding his/her answers. The Show Answer button immediately presents answers to the questions and the Explain button can be used for review or clarification purposes. It presents the student with a brief explanation of the appropriate method and/or theory.

- Precision and speed of diagrams and calculations are guaranteed for students and teachers working in *Math Trek High School*. This advantage permeates every section.

- *Math Trek High School* technology also provides positive alternatives for students with special needs. Individualized programs, access to calculators, modified workload, extra drill and examples are all available in *Math Trek High School*. An Audio button provides an audio version of the text displayed on screen. This feature removes barriers for students with reading difficulties and can also help improve their reading skills. Inherent in the program are structures that provide for independence, self-assessment, self-motivation, and flexibility of topic choice and depth. Both teacher and student can monitor progress and pace. Class performance profiles are available.

- The Journal option in the Tools button provides the students with a modified word processor. Here students have the opportunity to express their questions, observations, explanations, or reactions as a self- or teacher-directed activity.

NCTM Standards

NCTM (National Council of Teachers of Mathematics) has developed ten Standards (five content and five process) that summarize and describe the mathematical understanding, knowledge and skills that students should acquire.

1. Number and Operations

Instructional programs should enable all students to:

- understand numbers, ways of representing numbers, relationships among numbers, and number systems;
- understand meanings of operations and how they relate to one another;
- compute fluently and make reasonable estimates.

2. Algebra

Instructional programs should enable all students to:

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts.

3. Geometry

Instructional programs should enable all students to:

- analyze characteristics and properties of two-and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships;
- specify locations and describe spatial relationships using coordinate geometry and other representational systems;
- apply transformations and use symmetry to analyze mathematical situations;
- use visualization, spatial reasoning, and geometric modelling to solve problems.

4. Measurement

Instructional programs should enable all students to:

- understand measurable attributes of objects and the units, systems, and processes of measurement;
- apply appropriate techniques, tools, and formulas to determine measurements.

5. Data Analysis and Probability

Instructional programs should enable all students to:

- formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them;
- select and use appropriate statistical methods to analyze data;
- develop and evaluate inferences and predictions that are based on data;
- understand and apply basic concepts of probability.

6. Problem Solving

Instructional programs should enable all students to:
- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

7. Reasoning and Proof

Instructional programs should enable all students to:
- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

8. Communication

Instructional programs should enable all students to:
- organize and consolidate their mathematical thinking through communication;
- communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- analyze and evaluate the mathematical thinking and strategies of others;
- use the language of mathematics to express mathematical ideas precisely.

9. Connections

Instructional programs should enable all students to:
- recognize and use connections among mathematical ideas;
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- recognize and apply mathematics in contexts outside of mathematics.

10. Representation

Instructional programs should enable all students to:
- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Implementation in the Classroom

INTRODUCTION

Math Trek High School is based on several fundamental principles of mathematics teaching and learning. Students learn mathematics:

- through effective, practical experiences,
- when actively studying new knowledge from experience and prior knowledge and understanding, (leaning mathematics is a cumulative process with interaction between the strands),
- when presented with appropriately selected challenges and tasks,
- through regular, positive feedback and assessment that focuses on both content and process,
- with the appropriate use of technology,
- at different paces and using different learning styles, (remediation and enrichment are provided for each strand),
- when presented with appropriate high expectations and sufficient support to achieve these expectations,
- through communication, both oral and written, and in open discussions of content, procedure, and problem solving,
- through problem solving when they are challenged to consolidate understanding and experiences in different topics to address real-life situations.

TEACHING/LEARNING STRATEGIES

Math Trek High School software can be used in several ways:

- to introduce a particular topic or concept;
- to address skill remediation by providing students with practice;
- to extend and enrich the mathematics program;
- to assess skill development and set future learning goals.

Whole Class Instruction

a. As the teacher plans the mathematics program, certain topics, skills, and concepts are identified as being effectively delivered through use of the software. Students proceed to complete tutorials and activities that the teacher identifies as part of an integrated classroom mathematics program.
b. The teacher uses the software in combination with a projection device as a tool to teach skills or, for example, to model the mathematical algorithms. Students, subsequently, proceed individually to complete activities and performance tasks.

Small Group Learning

a. Students work in pairs to complete the tutorials. Activities may be completed in partners or individually.
b. Students in a group work on the same section and then proceed to complete the practice or exercises individually.

Individual Learning

Students complete the tutorials, activities, and performance tasks on an individual basis. There may be considerable diversity in where the students are working within the software. Student-teacher conferences are held to guide and monitor progress.

Certain students are directed to specific sections of the software for purposes of remediation and/or practice.

Certain students are directed to specific sections of the software for enrichment.

ASSESSMENT AND EVALUATION

Approach

Assessment should include a variety of techniques and instruments. The overall goal is to improve student learning. Within the software there are multiple opportunities for educators to use assessment to this end. The computer-assessed activities and quizzes provide one dimension to assessment in that the student receives immediate feedback and is fully aware of areas requiring improvement. Multiple-choice questions and short answer items assist to determine students' acquisition of specific skills and knowledge. However, these types of questions assess only one aspect of the problem – solving process.

Students are able to demonstrate their ability to carry out the complete problem-solving process through performance-based assessments that focus on higher level thinking skills. Research indicates that both instruction and assessment methods, which incorporate contextualized performance-based tasks, result in improved student learning. For this reason, samples of performance-based assessments that compliment *Math Trek High School* software, are included in the manual.

Student Tracking System

Software tracking is an integral feature of *Math Trek High School*. The teacher can obtain a listing of the sections of the software completed by a student. Quiz and activity results are scored within the tracking system. The teacher can access records of individual students and entire classes.

Techniques and Instruments

Performance Tasks: These authentic challenges result in a production or performance within a real-world context. Sample rubrics that can be used with the performance tasks are found in the manual.

Scoring Rubrics: These tools identify and describe criteria; student work is assessed according to the descriptors in levels of achievement. Should a 'mark' need to be assigned, the particular level corresponds with a numeric value.

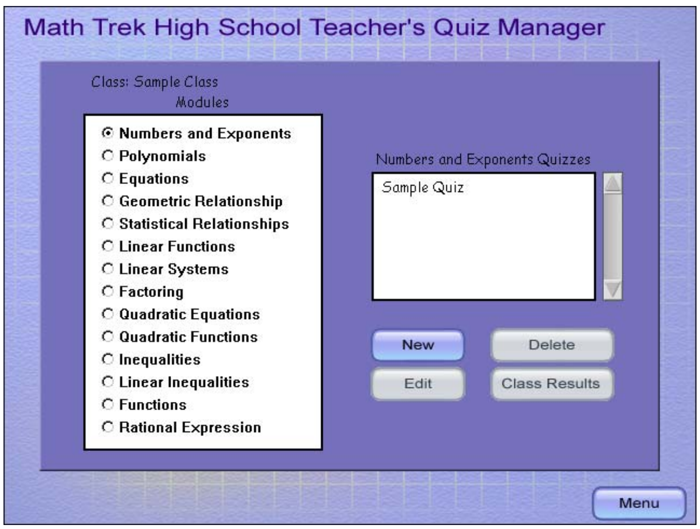
Checklists: These assist students in self-assessment.

Examples of instruments are included.

Quizzes

Each section contains a Quiz topic. After selecting this option, the student has three choices available.

1. Practice: Students select the questions for practice. Each question is scored with a show answer and an Explain feature available for incorrect answers.
2. Sample Quiz: A set number of questions is provided; however, the values in the questions are randomized. Therefore, the sample quiz can be retaken. Student scores are tracked.
3. Teacher Quiz: The teacher must pre-select the questions and save a quiz before students can access a Teacher Quiz. The values in the questions are randomized. Student scores are tracked.



Teacher Quiz Screen

Math Trek High School Journal

Communication of mathematical thinking confirms understanding of specific concepts and skills. In *Math Trek High School* numerous opportunities exist for students to respond in writing to generic questions:

1. Explain your solution to this problem.
2. Consider alternate solutions.
3. Pose another question of this type.
4. Suggest other examples.
5. Justify your choices.
6. Develop other puzzles.
7. Summarize your conclusions. (after using a mathematical explorer)
8. Follow your mathematical reasoning in the solution to this problem.
9. Provide direction in the solution to this problem. (after examining another student's work that contained errors or incomplete solutions)

Specific opportunities to use the journal are provided in these sections of the software:

Topic	Subsection
Numbers	Negative/zero Exponents
Equations	Solving Equations
Geometric Relationships	Angles
Statistical Relationships	Collecting Data Analyzing Data Line of Best Fit
Rational Expressions	Simplifying

Connections to Real Life

Through connections to real life activities and applications, students are engaged in mathematics and see relevance in what they are learning. Specific sections of the software that provide explicit connections are:

Topic	Subsection
Numbers	Integers Evaluating Powers Scientific Notation
Polynomials	Introduction Adding and Subtracting
Equations	Introduction Solving Equations Formulas
Geometric Relationships	Surface Area Volume
Statistical Relationships	Collecting Data Organizing Data Analyzing Data Line of Best Fit
Linear Functions	Introduction Slopes and Intercepts Alternate Forms of Linear Equations Interpolation and Extrapolation Application
Linear Systems	Introduction Graphs Substitution Elimination Applications
Quadratic Equations	Introduction Completing the Square Quadratic Formula Applications
Quadratic Functions	Curve of Best Fit Applications
Inequalities	Solving Inequalities
Linear Inequalities	Application
Rational Expressions	Introduction

MATHEMATICS EXPLORERS

Explorers are used to investigate dynamic relationships and to facilitate the use of deductive reasoning. Students can explore how changes in values impact graphs, diagrams, geometric figures, and formulas. From their exploration they can use the higher level thinking skills, such as analysis, synthesis, and evaluation to arrive at conclusions for further mathematical testing and justification. In *Math Trek High School*, mathematical explorers are used in these sections:

Topic	**Subsection**	**Screen Numbers**
Numbers	Squares and Square Roots Evaluating Powers Exponential Laws Negative/zero Exponents Scientific Notation	5 5 2, 11, 22 2, 6, 9 5, 16
Geometric Relationships	Angles Triangles Polygons Volume	2, 3, 9-12 1, 3, 4, 6, 7, 9-11 2-7 9, 10, 19
Statistical Relationships	Organizing Data Analyzing Data Line of Best Fit	15 5 2
Linear Functions	Introduction Slopes and Intercepts	5 6, 9
Quadratic Functions	Introduction Graphing	2 3, 6, 8, 11
Functions	Domain Range	9, 11 8

Mathematics Assessment Rubric

Level	1	2	3	4
Mathematical Analysis	Mathematical analysis was involved to a limited extent in student's work.	Mathematical analysis was involved in some portion of the student's work.	Mathematical analysis was involved in a significant proportion of the student's work.	Mathematical analysis was involved throughout the student's work.
Mathematical Concepts	The student demonstrates little understanding of the mathematical concepts that are central to the task, i.e., does not go beyond mechanical application of an algorithm.	The student demonstrates some understanding of the mathematical concepts that are central to the task. Where the student uses appropriate mathematical concepts, the application is incomplete.	There is substantial evidence the student understands the mathematical concepts that are central to the task and applies these concepts to the task appropriately; however, there may be some minor flaws in their application, or details may be missing.	The student demonstrates exemplary understanding of the mathematical concepts that are central to the task. Their application is appropriate, flawless, and elegant.
Written Communication	Mathematical explanations, arguments, or representations are incomplete, inappropriate, or incorrect.	Mathematical explanations, arguments, or representations are present. However, they may not be finished, may omit a significant part of an argument/explanation, or may contain significant mathematical errors.	Mathematical explanations or arguments are present. They are reasonably clear and accurate.	Mathematical explanations or arguments are clear, convincing, and accurate, with no significant mathematical errors.

Rubric for Performance-based Assessment

Category	Criteria	1	2	3	4
Knowledge/ Understanding	- understanding of concepts - performing algorithms, operations and procedures	demonstrates limited understanding of concepts and performs simple algorithms and procedures with limited accuracy	demonstrates limited understanding of concepts and performs simple algorithms and procedures with accuracy	demonstrates some understanding of concepts and performs algorithms and procedures with accuracy most of the time	demonstrates considerable understanding of concepts and performs algorithms and procedures with consistent accuracy
Application	- selecting concepts, formulas, algorithms, tools to fit the information given	seldom selects concepts, formulas, algorithms, tools and/or procedures to make connections with the information given	selects simple concepts, formulas, algorithms, tools and procedures and makes simple connections with the information given illustrating some understanding of the simple context	selects some concepts, formulas, algorithms, tools and procedures and makes general connections with the information given illustrating considerable understanding of the mathematical context	selects appropriate concepts, formulas, algorithms, tools and procedures and makes complex connections with the information given illustrating thorough understanding of the mathematical context
Thinking/Inquiry/ Problem Solving	- reasoning - applying the steps of an inquiry/problem-solving process	works through the questions with limited evidence of planning or forming a hypothesis; applies steps of an inquiry/ problem-solving process with some effectiveness; draws conclusions with very little supporting evidence and offers an incomplete summary	plans or forms a hypothesis that connects several aspects of the problem; applies the steps of an inquiry/problem-solving process with some effectiveness; draws conclusions with very little supporting evidence and offers an incomplete summary	plans or forms a hypothesis that connects several aspects of the problem; applies the steps of inquiry/ problem-solving process with moderate effectiveness; draws conclusions consistent with the work done and offers a summary of process used with no generalizations	plans or forms a hypothesis that connects aspects of the problem; applies the steps of an inquiry/ problem-solving process with considerable effectiveness; draws appropriate conclusions that are supported with evidence and offers a summary of process used with generalizations
Communication	- explaining reasoning - uses mathematical language, symbols, visuals and conventions	communicates with limited clarity and uses mathematical expressions and conventions inconsistently	communicates with some clarity and limited justification of reasoning and frequently uses mathematical expressions and conventions correctly	communicates explanations and justifications with considerable clarity and uses mathematical expressions and conventions correctly with some consistency	communicates explanations and justifications with a high degree of clarity and uses mathematical expressions and conventions correctly most of the time

Criteria Rubric for Assessment in Mathematics

Area	Criteria	Level 1	Level 2	Level 3	Level 4
Knowledge	- understanding of concepts	limited understanding of concepts	some understanding of concepts	considerable understanding of concepts	high degree of understanding of concepts
Problem Solving	- understanding of problems	needs assistance to understand problems	partially understands problem	needs little assistance to understand problem	completely understands problem
	- strategies for problem solving	limited ability to select strategies	some ability to select strategies	selects strategies independently	selects strategies effectively and independently
	- thinking skills	makes unsupported statements and/or illogical conclusions	supports statements to some extent and draws somewhat logical conclusions	supports statements and draws logical conclusions	thoroughly supports statements and draws insightful conclusions
Application	- skills	inconsistently applies skills	applies skills with some consistency	applies skills with considerable consistency	creatively and effectively applies skills
	- accuracy	major errors and/or omissions	few major errors/omission	some minor error/omissions	few, or no omissions and/or errors
	- difficulty of task	displays accuracy on simple tasks	displays accuracy on tasks on moderate complexity	displays accuracy on tasks of considerable complexity	displays accuracy and insight on considerably complex tasks
Communication	- reading	requires some assistance to interpret questions	requires minimal assistance to interpret questions	interprets questions independently	interprets complex questions independently
	- questioning	clarifies questions to a limited extent	clarifies questions to some extent	clarifies questions to a considerable extent	clarifies questions thoroughly
	- explaining	incomplete explanation of answer/solution	somewhat complete explanation of answer/solution	complete explanation of answer/solution	thorough explanation of answer/solution
	- reporting	communicates results to a limited extent	communicates results to some extent	communicates results to a considerable extent	communicates results to a thorough extent

Class Observation Checklist

Code: A = B = C = D = Date:____________________ Student's Name	Makes convincing argument	Contributes information or ideas	Applies a variety of problem solving strategies	Formulates clarifying questions	Modifies views when faced with new or conflicting evidence	Considers facts before reaching conclusions	Reflects on the reasonable-ness of answer	Supports ideas and observations with facts or details	Analyzes a problem

Self Assessment
Problem Solving Approach

As you reflect on the process, use #'s 1 – 4 to describe your performance. Explain each rating.

1 I needed assistance	2 I needed hints	3 I worked it out on my own	4 I solved problem without hesitation

Reflect **Activity/Comments**:			
'Wonder' Phase: Did I list what I knew from the problem?			
Did I go get other information to help me?			
Did I think about any assumptions I was making?			
Did I think about how to organize the information?			
Did I restate the problem in my own words?			
Did I wonder about a strategy to use?			
Working Phase: Did I try a variety of strategies?			
Did I look for a pattern?			
If I was frustrated, did I go back and look at the problem?			
If I tried a strategy and it didn't work, did I try it again to see if the same would happen?			
Did I make errors in calculations?			
Review Phase: Does by solution make sense?			
Can I justify my solution?			
Did I communicate my solution effectively in writing?			
Did I look to see if there was a different solution?			

Rubric for Mathematics Assessment

Category	Criteria	1	2	3	4
Knowledge/ Understanding	- understanding of concepts	demonstrates limited understanding of concepts	demonstrates some understanding of concepts	demonstrates considerable understanding of concepts	demonstrates thorough understanding of concepts
	- performing algorithms,	performs only simple algorithms accurately by hand and using technology	performs algorithms with inconsistent accuracy by hand, mentally, and using technology	performs algorithms accurately by hand, mentally, and using technology	selects the most efficient algorithm and performs it accurately by hand, mentally, and using technology
Thinking/Inquiry/ Problem Solving	- reasoning	follows simple mathematical arguments	follows arguments of moderate complexity and makes simple arguments	follows arguments of considerable complexity, judges the validity of arguments, and makes arguments of some complexity	follows complex arguments judges the validity of arguments, and makes complex arguments
	- applying the steps of an inquiry/problem-solving process	applies the steps of an inquiry/ problem-solving process with limited effectiveness	applies the steps of an inquiry/problem-solving process with moderate effectiveness	applies the steps of an inquiry/problem-solving process with considerable effectiveness	applies the steps of an inquiry/ problem-solving process with a high degree of effectiveness and poses extending questions
Communication	- communicating reasoning orally, in writing, and graphically	communicates with limited clarity and limited justification of reasoning	communicates with some clarity and some justification of reasoning	communicates with considerable clarity and considerable justification of reasoning	communicates concisely with a high degree of clarity and full justification of reasoning
	- use of mathematical language, symbols, visuals, and conventions	infrequently uses mathematical language, symbols, visuals and conventions correctly	uses mathematical language, symbols, visuals, and conventions correctly some of the time	uses mathematical language, symbols, visuals, and conventions correctly most of the time	routinely uses mathematical language, symbols, visuals, and conventions correctly and efficiently
Application	- applying concepts and procedures relating to familiar and unfamiliar settings	applies concepts and procedures to solve simple problems relating to familiar settings	applies concepts and procedures to solve problems of some complexity relating to familiar settings	applies concepts and procedures to solve complex problems relating to familiar settings; recognizes major mathematical concepts and procedures relating to applications in unfamiliar settings	applies concepts and procedures to solve complex problems relating to familiar and unfamiliar settings

Learning Outcomes

These charts delineate the learning outcomes and identify the specific sections of *Math Trek High School* where the skills and knowledge are addressed and assessed.

NUMBERS

Sections:

1. Integers
2. Rational Numbers
3. Squares and Square Roots
4. Evaluating Powers
5. Exponent Laws
6. Negative/Zero Exponents
7. Scientific Notation

Code	Skills	Section						
		1	2	3	4	5	6	7
N1	identify and use the arithmetic properties of subsets of integers and rational, irrational, and real numbers, including closure properties for the four basic arithmetic operations where applicable	√	√	√				
N2	use properties of numbers to demonstrate whether assertions are true or false					√		
N3	express the square root of a whole number in simplest radical form and approximate square roots to the nearest tenth			√				
N4	estimate the square root of a non-perfect square to the nearest tenth by identifying the two perfect squares it lies between; finding the square root of those two perfect squares; and using those values estimate the square root of the non-perfect square			√				
N5	find the square root of a number, and make a reasonable interpretation of the displayed value for a given situation, using a calculator			√				
N6	express the square root of a whole number less than 1000 in simplest radical form			√				
N7	evaluate numerical expressions involving natural number exponents with rational number bases				√	√	√	
N8	determine the meaning of negative exponents and of zero as an exponent					√	√	
N9	represent very large and very small numbers, using scientific notations							√
N10	enter and interpret exponential notation on a scientific calculator, as necessary in calculations involving very large and very small numbers							√
N11	determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials and the exponent rule for the power of a power, and apply these rules in expressions involving one and two variables					√	√	√
N12	use patterns to generate the laws of exponents and apply them in problem-solving situations					√	√	√
N13	express numbers, using scientific notation, and perform operations, using the laws of exponents							√
N14	apply the laws of exponents to perform operations on expressions with integral exponents, using scientific notation when appropriate							√
N15	convert a rational number to a decimal number	√						
N16	compute the square of a real number			√				
N17	solve real life problems involving exponential growth					√		

POLYNOMIALS

Sections:

1. Introduction
2. Adding and Subtracting
3. Multiplying Polynomials

Code	Skills	Section		
		1	2	3
P1	represent verbal quantitative situations algebraically and evaluate these expressions for given replacement values of the variables	√	√	
P2	translate verbal expressions into algebraic expressions with three or fewer terms	√	√	
P3	relate a polynomial expression with three or fewer terms to a verbal expression	√	√	
P4	substitute into and evaluate algebraic expressions involving exponents, to support other topics	√		
P5	evaluate algebraic expressions for a given replacement set to include integers and rational numbers	√		
P6	apply appropriate computational techniques to evaluate an algebraic expression	√		
P7	use the commutative, associative, and distributive properties to simplify algebraic expressions			√
P8	simplify monomial expressions and ratios of monomial expressions in which the exponents are integers, using the laws of exponents			√
P9	add, subtract, and multiply monomials and polynomials; solve multi-step problems, including work problems, by using these techniques		√	√
P10	add and subtract polynomials		√	
P11	expand and simplify polynomial expressions involving one variable		√	√
P12	add, subtract, and multiply polynomials, using concrete objects, pictorial and area representations, and algebraic manipulations		√	√
P13	model sums, differences, and products of polynomials with concrete objects and their related pictorial representations		√	√
P14	relate concrete and pictorial representations for polynomial operations to their corresponding algebraic manipulations		√	√
P15	find sums and differences of polynomials		√	
P16	multiply polynomials by monomials and binomials by binomials symbolically			√

EQUATIONS

Sections:

1. Introduction
2. Solving Equations
3. Formulas

Code	Skills	Section		
		1	2	3
E1	identify the corresponding mathematical symbol, expression, or equation for a verbal description	√		
E2	solve first degree equations, including equations with fractional coefficients, using an algebraic method		√	√
E3	rearrange formulas involving variables in the first degree, with and without substitution		√	√
E4	solve a literal equation (formula) for a specified variable			√
E5	apply skills for solving linear equations to practical situations			√
E6	solve equations, using the addition, multiplication, closure, identity, and inverse properties		√	√
E7	solve equations, using the reflexive, symmetric, transitive, and substitution properties of equality		√	√
E8	translate verbal sentences to algebraic equations in one variable	√		
E9	solve multi-step linear equations in one variable with the variable in both sides of the equation		√	

E10	solve multi-step linear equations in one variable with grouping symbols in one or both sides of the equation		√	
E11	solve multi-step equations in one variable with rational coefficients and constants		√	

GEOMETRIC RELATIONSHIPS

Sections:

1. Angles
2. Triangles
3. Polygons
4. Surface Area
5. Volume

Code	Skills	Section				
		1	2	3	4	5
GR1	illustrate and explain the properties of the interior and the exterior angles of triangles, quadrilaterals, and other polygons, and of angles related to parallel lines	√	√			
GR2	apply the relationships between the measures of angles formed by a transversal	√				
GR3	determine and apply the properties of angle bisectors, medians, centroids, incenters, and altitudes in various types of triangles through investigation		√			
GR4	determine and apply the properties of the sides and the diagonals of polygons through investigation			√		
GR5	pose questions about geometric relationships, test them, and communicate the findings, using appropriate language and mathematical forms	√	√	√	√	√
GR6	confirm a statement about the relationships between geometric properties by illustrating the statement with examples	√	√	√	√	√
GR7	use graphs to represent nonlinear relations derived from descriptions of realistic situations					√
GR8	recognize the 3-dimensional shape that can be formed from a given 2-dimensional net				√	
GR9	determine the area of triangles, rectangles, circles				√	
GR10	identify, through investigation, the effect of varying the dimensions of a rectangular prism or cylinder on the volume or surface area of the object				√	√
GR11	identify, through investigation, the relationships between the volume and surface area of a given rectangular prism or cylinder				√	√
GR12	explain the significance of optimal surface area or volume in various applications				√	√
GR13	solve a problem involving the relationship between the perimetre and the area of a figure when one of the measures is fixed				√	
GR14	solve simple problems, using the formulas for the surface area and the volume of prisms, pyramids, cylinders, cones, and spheres				√	√
GR15	solve multi-step problems involving the volume and the surface area of prisms, cylinders, pyramids, cones, and spheres				√	√

STATISTICAL RELATIONSHIPS

Sections:

1. Collecting Data
2. Organizing Data
3. Analyzing Data
4. Line of Best Fit

Code	Skills	Section			
		1	2	3	4
SR1	demonstrate an understanding of some principles of sampling and surveying, and apply the principles in designing and carrying out experiments to investigate the relationships between variables	√	√	√	
SR2	collect data, using appropriate equipment and/or technology		√	√	
SR3	organize and analyze data using appropriate techniques and technology		√	√	
SR4	describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and the hypotheses	√	√	√	√

SR5	communicate the findings of an experiment clearly and concisely, using appropriate mathematical forms	√	√		
SR6	construct tables of values and scatter plots for linearly related data collected from experiments or from secondary sources		√		
SR7	determine the equation of a line of best fit for a scatter plot, using an informal process				√
SR8	construct tables of values and scatter plots for nonlinearly related data collected from experiments (e.g., the relationship between height and age) or from secondary sources (e.g., the population of Canada over time)		√		
SR9	gather and record data, or use data sets, to determine functional (systematic) relationships between quantities	√	√	√	
SR10	in solving problems, collect and organize data, make and interpret scatterplots, and model, predict, and make decisions and critical judgments	√	√	√	
SR11	use matrices to organize and manipulate data, including matrix addition, subtraction, and scalar multiplication			√	
SR12	represent data from practical problems in matrix form		√	√	
SR13	calculate the sum or difference of two given matrices that are no larger than 4 x 4		√	√	
SR14	calculate the product of a scalar and a matrix that is no larger than 4 x 4		√	√	
SR15	solve practical problems involving matrix addition, subtraction, and scalar multiplication, using matrices that are no larger than 4 x 4		√	√	
SR16	read and interpret the data in a matrix representing the solution to a practical problem		√	√	
SR17	made predictions about unknown outcomes, using the equation of a line of best fit				√
SR18	compare and contrast multiple one-variable data sets, using statistical techniques that include measures of central tendency, range, and box-and-whisker graphs			√	
SR19	calculate the measures of central tendency and range of a set of data with no more than 20 data points			√	
SR20	compare measures of central tendency using numerical data from a table with no more than 20 data points			√	
SR21	compare and contrast two sets of data, each set having no more than 20 data points, using measures of central tendency and the range			√	
SR22	compare and analyze two sets of data, each set having no more than 20 data points, using box-and-whisker plots		√	√	

LINEAR FUNCTIONS

Sections:

1. Introduction
2. Linear and Non-linear Functions
3. Slope and Intercepts
4. Graphing Slope/Intercept Linear Functions
5. Alternate Forms of Linear Equations
6. Interpolation and Extrapolation
7. Applications

Code	**Skills**	**Section**						
		1	**2**	**3**	**4**	**5**	**6**	**7**
LF1	graph a linear equation and compute the x- and y-intercepts		√	√	√			
LF2	verify that a point lies on a line, given an equation of the line; derive linear equations by using the point-slope formula		√	√	√			
LF3	understand the concepts of parallel lines and perpendicular lines and how those slopes are related			√				
LF4	determine values of a linear relation by using the formula of the relation and by interpolating or extrapolating from the graph of the relation		√				√	
LF5	explore a situation that would explain the events illustrated by a given graph of a relationship between two variables							√

LF6	identify, by calculating finite differences in its table of values, whether a relation is linear or nonlinear		√					
LF7	investigate the effect on the graph and the formula of a relation of varying the conditions of a situation they represent		√				√	
LF8	determine, through investigations, the characteristics that distinguish the equation of a straight line from the equations of nonlinear relations		√	√		√		
LF9	select the equations of straight lines from a given set of equations of linear and nonlinear relations		√					
LF10	identify the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$			√	√	√		
LF11	convert a linear equation from standard form to slope-intercept form					√		
LF12	determine the slope of a line segment, using various formulas			√	√			
LF13	identify the slope of a linear relation as representing a constant rate of change			√	√			
LF14	calculate the finite differences in the table of values of a linear relation and relate the result to the slope of the relation		√					
LF15	identify the geometric significance of m and b in the equation $y = mx + b$ through investigation			√	√			
LF16	Identify the properties of the slopes of line segments through investigations facilitated by graphing technology			√	√			
LF17	plot points on the xy plane and use the terminology and notation of the xy plane correctly	√		√	√			
LF18	graph lines using a variety of techniques	√	√	√	√			
LF19	determine the equation of a line, given information about the line		√		√	√		
LF20	Investigate the concepts of the slope and y-intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation			√	√		√	
LF21	solve problems involving a situation that would be modelled by a given linear equation		√					√
LF22	construct tables of values, graphs, and formulas to represent linear relations derived from descriptions of realistic situations	√					√	√
LF23	demonstrate an understanding that straight lines represent linear relations and curves represent nonlinear relations		√					
LF24	understand that linear functions can be represented in different ways and translate among their various representations		√	√	√	√		
LF25	determine whether or not given situations can be represented by linear functions		√					
LF26	determine the domain and range values for which linear functions make sense for given situations	√						
LF27	translate among the use algebraic, tabular, graphical, or verbal descriptions of linear functions	√	√	√	√	√		
LF28	understand the meaning of the slope and intercepts of linear functions and interpret and describe the effects of changes in parametres of linear functions in real-world and mathematical situations			√	√			√
LF29	develop the concept of slope as rate of change and determine slopes from graphs, tables, and algebraic representations			√	√			
LF30	interpret the meaning of slope and intercepts in			√	√			

	situations using data, symbolic representations, or graphs							
LF31	investigate, describe, and predict the effects of changes in m and b on the graph of y = mx + b			√	√			
LF32	graph and write equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept	√		√	√	√		
LF33	determine the intercepts of linear functions from graphs, tables, and algebraic representations			√	√			
LF34	interpret and predict the effects of changing slope and y-intercept in applied situations			√	√			
LF35	relate direct variation to linear functions and solves problems involving proportional change			√				
LF36	identify linear equations that represent a pattern in which there is a constant rate of change between variables		√	√	√	√		
LF37	analyze a relation to determine whether a direct variation exists and represent it algebraically and graphically		√		√			
LF38	given a table of values, determine whether a direct variation exists			√				
LF39	graph a direct variation from a table of values or a practical situation				√			√
LF40	determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line; describe as rate of change and will be positive, negative, zero, or undefined			√	√			
LF41	recognize that m represents the slope in the equation of the form y = mx + b			√	√			
LF42	find the slope of the line, given the equation of a linear function			√	√			
LF43	calculate the slope of a line, given the coordinates of two points on the line			√	√			
LF44	find the slope of a line, given the graph of a line			√	√			
LF45	recognize and describe a line with a slope that is positive, negative, zero, or undefined			√				
LF46	describe slope as a constant rate of change between two variables			√				
LF47	compare the slopes of graphs of linear functions			√				
LF48	recognize that equations of the form y = mx + b and Ax + By = C are equations of lines				√			
LF49	identify an equation of a line when given the graph of a line				√	√		
LF50	identify an equation of a line when given two points on the line whose coordinates are integers				√	√		
LF51	write an equation of a line when given the slope and a point on the line whose coordinates are integers				√	√		
LF52	write an equation of a vertical line as x = c					√		
LF53	write an equation of a horizontal line as y = c					√		

LINEAR SYSTEMS

Sections:

1. Introduction
2. Graphs
3. Substitution
4. Elimination
5. Application

Code	Skills	Section				
		1	2	3	4	5
LS01	construct tables of values, graphs, and formulas to represent linear relations derived from descriptions of realistic situations	√	√			
LS02	solve a system of equations using a table of values	√				
LS03	solve a system of equations using graphing		√			
LS04	determine the number of solutions for a system of equations by analyzing the slopes of lines		√			
LS05	solve a system of equations using the method of substitution			√		
LS06	solve a system of equations using the method of elimination				√	
LS07	solve a system of equations in an applied situation					√
LS08	given a system of two linear equations, solve the system by substitution or elimination to find the ordered pair that satisfies both equations			√	√	
LS09	solve a system of two linear equations algebraically and interpret the answer graphically		√	√	√	√
LS11	determine whether a system of two linear equations has one solution, no solution, or infinite solutions		√			
LS12	interpret the reasonableness of the algebraic or graphical solution of a system of two linear equations that describes a practical situation		√			√
LS13	formulate systems of linear equations from problem situations					√

FACTORING

Sections:

1. Introduction
2. Greatest Common Factor
3. Difference of Squares
4. Perfect Squares
5. Simple Trinomials
6. Complex Trinomials
7. Multi-step Factoring

Code	Skills	Section						
		1	2	3	4	5	6	7
FA1	apply basic factoring techniques to second and simple third-degree polynomials, including finding a common factor for all terms in a polynomial, recognizing the difference of two squares, and recognizing perfect squares of binomials	√	√	√	√	√		
FA2	factor completely first and second degree binomials and trinomials in one or two variables	√	√	√	√	√	√	√
FA3	use the distributive property to “factor out” all common monomial factors	√	√					√
FA4	factor second degree polynomials and binomials with integral coefficients and a positive leading	√	√	√	√	√	√	√

	coefficient less than four							
FA5	identify polynomials that cannot be factored over the set of real numbers							√
FA6	factor a polynomial by removing a common factor	√	√					√

QUADRATIC EQUATIONS

Sections:

1. Introduction
2. Solving by Isolation
3. Solving by Factoring
4. Solving by Completing the Square
5. Quadratic Formula
6. Applications

Code	Skills	Section					
		1	2	3	4	5	6
QE01	identify whether or not an equation is a quadratic equation	√					
QE02	identify the root of a quadratic equation given a table of values	√					
QE03	solve a quadratic equation using the method of isolation		√				
QE04	solve a quadratic equation using factoring			√			
QE05	solve a quadratic equation using the quadratic formula					√	
QE06	evaluate the discriminant of a quadratic expression					√	
QE07	understand there is more than one way to solve a quadratic equation	√	√	√	√	√	√
QE08	determine the appropriate method to solve a quadratic equation						√
QE09	solve quadratic equations using tables, graphs, and algebraic methods	√	√	√			
QE10	solve a quadratic equation by factoring or completing the square			√	√		
QE11	understand and use the quadratic formula					√	
QE12	use the quadratic formula to find the roots of a second-degree polynomial					√	
QE13	apply quadratic formula to real life situations						√

QUADRATIC FUNCTIONS

Sections:

1. Introduction
2. Graphs
3. Curve of Best Fit
4. Applications

Code	Skills	Section			
		1	2	3	4
QF01	given a variety of quadratic graphs determine the value of one or more unknowns	√			
QF02	determine realistic values for the domain of a quadratic function				√
QF03	determine realistic values for the range of a quadratic function				√
QF04	determine the equation of a curve of best fit			√	
QF05	extrapolate using a curve of best fit			√	

QF06	describe the effect on the graph and the formula of a relation of varying conditions of a situation they represent	√	√		
QF07	graph quadratic functions and know that their roots are the x-intercepts		√		
QF08	use the quadratic formula or factoring techniques to determine whether the graph of a quadratic function will intersect the x-axis in zero, one, or two points		√		
QF09	apply quadratic functions to physical problems		√		√
QF10	understand that the graphs of quadratic functions are affected by the coefficients of the function		√		
QF11	interpret the effects of changes in the oefficients of quadratic functions		√		
QF12	determine the domain and range values for which quadratic functions make sense for given situations				√
QF13	investigate, describe, and predict the effects of changes on a variety of quadratic graphs		√		
QF14	analyze graphs of quadratic functions and draw conclusions				√
QF15	relate the solutions of quadratic equations to the roots of their functions			√	
QF16	identify the x-intercepts of the quadratic function as the solution(s) to the quadratic equations that is formed by setting the given quadratic expression equal to zero		√		
QF17	apply algebraic techniques to solve rate problems and work problems				√

INEQUALITIES

Sections:

1. Introduction
2. Solving Equations

Code	Skills	Section	
		1	2
I 2	solve multi-step noncompound inequalities in one variable with the variable in both sides of the inequality		√
I 3	solve multi-step linear inequalities in one variable with grouping symbols in one or both sides of the inequality		√
I 4	solve multi-step inequalities in one variable with rational coefficients and constants		√
I 7	solve multi-step problems, including word problems, involving linear inequalities in one variable and provide justification for each step		√
I 8	justify steps used in simplifying expressions and solving inequalities		√

LINEAR INEQUALITIES

Sections:

1. Graphs
2. Systems
3. Applications

Code	Skill	Section		
		1	2	3
LI01	graph a linear inequality	√		
LI02	graph a system of linear inequalities	√		
LI03	solve a system of linear inequalities in an applied situation		√	
LI04	understand the events illustrated by a given graph of a relationship between two variables		√	√
LI05	interpret and determine the reasonableness of solutions to systems of linear equations			√
LI06	formulate inequalities based on linear functions	√	√	√
LI07	use a variety of methods to solve inequalities based on linear functions and analyze the solutions	√	√	√
LI08	analyze situations involving linear functions and formulate inequalities to solve problems			√
LI09	investigate methods for solving inequalities using graphs and the properties of inequality	√	√	
LI10	select, justify, and apply an appropriate technique to graph linear inequalities	√	√	√
LI011	graph linear inequalities that arise from a variety of practical situations	√	√	√

FUNCTIONS

Sections:

1. Introduction
2. Domain
3. Range
4. Values and Zeros

Code	Skills	Section			
		1	2	3	4
F1	understand the concepts of a relation and a function, determine whether a given relation defines a function, and give pertinent information about given relations and functions	√	√	√	√
F2	determine the domain of independent variables and the range of dependent variables defined by a graph, a set of ordered pairs, or a symbolic expression		√	√	
F3	determine whether a relation defined by a graph, a set of ordered pairs, or a symbolic expression is a function	√			
F4	understand that a function represents a dependence of one quantity or another and can be described in a variety of ways	√	√	√	
F5	describe independent and dependent quantities in functional relationships	√	√	√	
F6	interpret and make inferences from functional relationships	√	√	√	√
F7	use the properties and attributes of functions	√	√	√	√
F8	identify the mathematical domains and ranges and determine reasonable domain and range values for given situations		√	√	
F9	create and use tabular, symbolic, graphical, verbal, and physical representations to analyze a given set of data for the existence of a pattern, determine the domain and range of relations, and identify the relations that are functions	√	√	√	
F10	analyze a table of ordered pairs for the existence of a pattern that defines the change relating input and output values	√	√	√	

F11	determine from a set of ordered pairs, a table, or a graph whether a relation is a function	√			
F12	identify the domain and range for a relation, given a set of ordered pairs, a table, or a graph		√	√	
F13	given a rule, fine the values of a function for elements in its domain and locate the zeros of the function both algebraically and graphically				√
F14	for each x in the domain of f, find (f(x)				√
F15	identify the zeros of the function algebraically and confirm them, using graphs				√

RATIONAL EXPRESSIONS

Sections:

1. Introduction
2. Stating Restrictions
3. Simplifying
4. Multiplying
5. Dividing
6. Adding and Subtracting

Code	**Skills**	**Section**					
		1	**2**	**3**	**4**	**5**	**6**
RE1	simplify expressions using the commutative, associative, and distributive properties and state any restrictions on the variables		√	√	√	√	√
RE2	simplify expressions using the order of operations and state any restrictions on the variables		√	√	√	√	√
RE3	interpret pictorial representations for simplifying expressions	√					
RE4	add, comma, multiply and divide rational expressions and functions				√	√	√
RE5	Solve problems by performing arithmetic operations on rational expressions and state any restrictions on the variables		√	√	√	√	√

Cross-References

Numbers Quiz Questions Descriptors

Integers

1. Compute the sum of a number of integers.
2. Compute the difference of two integers.
3. Compute the product of two integers.
4. Compute the quotient of two integers.

Rational Numbers

1. Compute the product of two rational numbers.
2. Compute the quotient of two rational numbers.
3. Compute the sum of two rational numbers.
4. Compute the difference of two rational numbers.
5. Convert a rational number to a decimal number.

Squares and Square Roots

1. Compute the square of a positive integer.
2. Compute the square of a negative integer.
3. Compute the square of a real number.
4. Compute the square root of a real number.
5. Compute the negative square root of a real number.
6. Estimate the square root of a rational number. Level of difficulty: 1.
7. Estimate the square root of a rational number. Level of difficulty: 2.
8. Reduce an entire radical to a mixed radical. Level of difficulty: 1.
9. Reduce an entire radical to a mixed radical. Level of difficulty: 2.

Evaluating Powers

1. Evaluate a power with a whole number base and exponent.
2. Evaluate a power with a fractional base and whole number exponent.
3. Evaluate a power with a decimal number base and whole number exponent.
4. Solve a real life problem involving exponential growth. Level of difficulty: 1.
5. Solve a real life problem involving exponential growth. Level of difficulty: 2.

Exponent Laws

1. Simplify the product of two powers with the same whole number base using the laws of exponents.
2. Simplify the product of two powers with the same variable base using the laws of exponents.
3. Simplify and evaluate the product of two powers with the same whole number base using the laws of exponents.
4. Express a power with an integer base as a product of two other powers using the laws of exponents.
5. Express a power with a variable base as a product of two other powers using the laws of exponents.
6. Simplify the quotient of two powers with the same integer base using the laws of exponents.
7. Simplify the quotient of two powers with the same variable base using the laws of exponents.
8. Simplify an expression involving multiplication and division of powers with the same integer base using the laws of exponents. Level of difficulty: 1.
9. Simplify an expression involving multiplication and division of powers with the same variable base using the laws of exponents. Level of difficulty: 1.
10. Simplify an expression involving multiplication and division of powers with the same integer base using the laws of exponents. Level of difficulty: 2.

11. Simplify an expression involving multiplication and division of powers with the same variable base using the laws of exponents. Level of difficulty: 2.
12. Simplify and evaluate an expression involving multiplication and division of powers with the same integer base using the laws of exponents.
13. Substitute values for variable and evaluate a monomial expression in a geometric context.
14. Simplify a power of a power with a whole number base using exponent laws.
15. Simplify a power of a power with a variable base using exponent laws.
16. Simplify and evaluate power of a power with a whole number base using the laws of exponents.
17. Use the exponent laws for power of a power with a whole number base to solve for a variable. Level of difficulty: 1.
18. Use the exponent laws for power of a power with a variable base to solve for a variable. Level of difficulty: 1.
19. Use the exponent laws for power of a power with a variable base to solve for a variable. Level of difficulty: 2.
20. Use the exponent laws for power of a power with a whole number base to solve for a variable.
21. Use the exponent laws for power of a power with a variable base to solve for a variable. Level of difficulty: 3.
22. Use exponent laws to simplify the product of two powers of a power with whole number bases to solve for a variable.
23. Use exponent laws to simplify the product of two powers of a power with variable bases to solve for a variable.
24. Simplify an expression involving multiplication and division of powers of powers using laws of exponents. Level of difficulty: 1.
25. Simplify an expression involving multiplication and division of powers of powers using laws of exponents. Level of difficulty: 2.
26. Simplify an expression involving multiplication and division of powers of powers using laws of exponents. Level of difficulty: 3.

Negative/Zero Exponents

1. Express a power with an integer base and a negative exponent as a power with a positive exponent.
2. Evaluate a power with an integer base and a negative exponent.
3. Evaluate the product of powers with whole number bases and negative exponents.
4. Simplify a quotient of powers with whole number bases and negative exponents.
5. Simplify and evaluate a quotient of powers with whole number bases involving two negative exponents.
6. Simplify and evaluate a quotient of powers with whole number bases involving one positive and one negative exponent.
7. Simplify and evaluate an expression containing multiplication and division of powers using laws of exponents. Level of difficulty: 1.
8. Simplify and evaluate an expression containing multiplication and division of powers using laws of exponents. Level of difficulty: 2 .
9. Simplify and evaluate an expression containing multiplication and division of powers using laws of exponents. Level of difficulty: 3.

Scientific Notation

1. Convert a number from scientific to standard form. Level of difficulty: 1.
2. Convert a number from scientific to standard form. Level of difficulty: 2.
3. Convert a number from standard to scientific form. Level of difficulty: 1.
4. Convert a number from standard to scientific form. Level of difficulty: 2.
5. Convert a number from standard to scientific form. Level of difficulty: 3.
6. Multiply two numbers in scientific form.
7. Multiply two numbers in standard form by converting them to scientific form.
8. Divide two numbers in scientific form in an applied context. Level of difficulty: 1.
9. Divide two numbers in scientific form in an applied context. Level of difficulty: 2.

10. Divide two numbers in scientific form in an applied context. Level of difficulty: 3.
11. Divide two numbers in scientific form.
12. Divide two numbers in standard form by converting them to scientific form.
13. Solve problems in an applied context using scientific notation. Level of difficulty: 1.
14. Solve problems in an applied context using scientific notation. Level of difficulty: 2.
15. Solve problems in an applied context using scientific notation. Level of difficulty: 3.
16. Solve problems in an applied context using scientific notation. Level of difficulty: 4.
17. Solve problems in an applied context using scientific notation. Level of difficulty: 5.
18. Solve problems in an applied context using scientific notation. Level of difficulty: 6.

Polynomials Quiz Questions Descriptors

Introduction

1. Calculate the missing quantity when provided with two of three quantities related by subtraction.
2. Calculate the missing quantity when provided with two of three quantities related by division.
3. Identify the corresponding mathematical symbol for a verbal description.
4. Identify the corresponding mathematical expression for a verbal description.
5. Substitute an integer for a variable in an expression and evaluate it.
6. Substitute decimal numbers for variables in an expression and evaluate it.
7. Substitute fractions for variables in an expression and evaluate it.
8. Identify monomials that are like terms.
9. Identify the polynomial represented by a given set of algebra tiles.
10. Distinguish between monomials, binomials and trinomials.

Adding and Subtracting

1. Identify a sum of polynomials represented by a set of algebra tiles.
2. Identify a difference of polynomials represented by a set of algebra tiles.
3. Simplify polynomials by combining like terms.
4. Add polynomials.
5. Add polynomials in an applied context.
6. Subtract polynomials.
7. Subtract polynomials in an applied context.
8. Add and subtract polynomials in an applied context. Level of difficulty: 1.
9. Add and subtract polynomials in an applied context. Level of difficulty: 2.
10. Add and subtract polynomials in an applied context. Level of difficulty: 3.
11. Add and subtract polynomials in an applied context. Level of difficulty: 4.

Multiplying

1. Recognize the result of applying the distributive property as represented by a set of algebra tiles.
2. Recognize the result of applying the distributive property as represented by algebra expressions.
3. Multiply two monomials.
4. Multiply two monomials in an applied context.
5. Multiply a binomial by a monomial. Level of difficulty: 1.
6. Multiply a binomial by a monomial. Level of difficulty: 2.
7. Multiply a binomial by a monomial in an applied context.
8. Multiply a trinomial by a monomial. Level of difficulty: 1.
9. Multiply a trinomial by a monomial. Level of difficulty: 2.
10. Multiply a trinomial by a monomial. Level of difficulty: 3.
11. Multiply two binomials by two monomials and add the result. Level of difficulty: 1.
12. Multiply two binomials by two monomials and subtract the result. Level of difficulty: 1.
13. Multiply two binomials by two monomials and add the result. Level of difficulty: 2.
14. Multiply two binomials by two monomials and subtract the result. Level of difficulty: 2.

15. Multiply two trinomials by two monomials and add the result.
16. Multiply two trinomials by two monomials and subtract the result.
17. Multiply two binomials.
18. Multiply a trinomial by a binomial.
19. Multiply two trinomials by two binomials and add the result.
20. Multiply two trinomials by two binomials and subtract the result.

Equations Quiz Questions Descriptors

Introduction

1. Identify the corresponding mathematical symbol for a verbal description.
2. Identify the corresponding mathematical expression for a verbal description.
3. Identify the corresponding mathematical equation for a verbal description. Level of difficulty: 1.
4. Identify the corresponding mathematical equation for a verbal description. Level of difficulty: 2.

Solving Equations

1. Use a flow chart to solve a linear equation.
2. Solve a linear equation algebraically. Level of difficulty: 1.
3. Solve a linear equation algebraically. Level of difficulty: 2.
4. Solve a linear equation algebraically. Level of difficulty: 3.
5. Solve a linear equation algebraically. Level of difficulty: 4.
6. Identify the lowest common denominator of a set of fractions.
7. Multiply a fraction by an integer.

Formulas

1. Use a formula to calculate the perimetre of a rectangle.
2. Use a formula to calculate the area of a rectangle.
3. Use a formula to calculate the circumference of a circle.
4. Use a formula to calculate the area of a circle.
5. Use a formula to calculate the area of a trapezoid.
6. Use a formula to calculate distance travelled.
7. Use a formula to calculate simple interest.
8. Use a formula to calculate the volume of a cylinder.
9. Use a formula to calculate electrical current.
10. Use a formula to convert a Celcius temperature to Fahrenheit.
11. Use a formula to convert a Fahrenheit temperature to Celcius.

Geometric Relationships Quiz Questions Descriptors

Angles and Polygons

1. Determine the measure of an interior angle of a polygon.
2. Apply the relationship between the interior and exterior angles of a triangle.
3. Determine the measures of the exterior angles of a polygon.
4. Distinguish between opposite, corresponding, alternate and co-interior angles.
5. Distinguish between congruent and supplementary angles.
6. Determine the sum of a pair of co-interior angles.
7. Apply the relationship between the measures of angles formed by 2 parallel lines and a transversal.

Triangles

1. Apply the relationship between the areas of the two triangles formed by a triangle's median.

2. Apply the relationship between a triangle's centroid and its median.
3. Apply the relationship between a triangle's incenter and its angle measures.

Polygons

1. Identify and distinguish between the various properties of quadrilaterals.
2. Determine the number of a diagonals of a polygon with n sides.

Surface Area

1. Determine the area of a triangle, rectangle, or circle.
2. Recognize the 3-dimensional shape that can be formed from a given 2-dimensional net.
3. Determine the surface area of a rectangular prism.
4. Determine the surface area of a triangular prism.
5. Determine the surface area of a cylinder.
6. Determine part of the surface area of a cylinder.
7. Determine the surface area of a rectangular prism from its net.
8. Determine part of the surface area of a cone.
9. Determine the surface area of a planet.
10. Determine the surface area of a composite shape.

Volume

1. Determine the volume of a rectangular prism.
2. Determine the volume of a triangular prism.
3. Determine the volume of a cylinder.
4. Determine the volume of a rectangular pyramid.
5. Determine the volume of a cone.
6. Determine the volume of a cone in an applied situation.
7. Determine the volume of a volleyball.
8. Determine the volume of a planet.
9. Determine the volume of a composite shape.

Statistical Relationships Quiz Questions Descriptors

Organizing Data

1. Identify the best methods for organizing data.
2. Identify the type of correlation shown by a scatter plot.

Analyzing Data

1. Calculate measures of central tendency.
2. Multiply a matrix by a scalar.
3. Add and subtract matrices.
4. Determine what matrix operations have no solution.

Linear Functions Quiz Questions Descriptors

Introduction

1. Identify the equation that corresponds to a given table of values.
2. Read the coordinates of a point from a Cartesian graph.
3. Determine the x-coordinate of an ordered pair of an equation given the y-coordinate.
4. Graph and determine the domain and range of a set of ordered pairs.

5. Complete a set of ordered pairs and graph a linear equation.

Linear vs. non-linear

1. Identify whether or not a function is linear or not given its graph.
2. Identify whether or not a function is linear or not by determining successive differences in a table of values.
3. Identify whether or not a given ordered pair lies on a given linear function.

Slope and intercepts

1. Calculate the slope of a line given two points on its graph.
2. Identify the slope of a line segment as positive, negative, zero, or undefined, given its graph.
3. Graph a line segment whose slope is positive, negative, zero, or undefined.
4. Identify the numerical relationship between the slopes of pairs of line segments that are perpendicular or parallel given their graphs.
5. Identify whether a relationship represents partial or direct variation given its graph.

Graphing

1. Determine the slope, y-intercept, and equation of a line given the coordinates of two points on the line.
2. Graph a line given its equation by determining the coordinates of two points on the line.
3. Graph a line given its equation by determining the coordinates of the x- and y-intercepts.
4. Graph a line given its slope and the coordinates of one point on the line.
5. Graph a line given its slope and y-intercept.

Alternate Forms

1. Graph equations of the form y = k.
2. Identify the equation of a horizontal line given its graph.
3. Match the equations of horizontal and vertical lines to their graphs.
4. Classify linear equations as having graphs that are either horizontal or vertical or neither
5. Convert a linear equation from standard form to slope-intercept form.
6. Identify corresponding standard form and slope-intercept form equations.

Applications

1. Graph a linear equation representing a real life situation.

Linear Systems Quiz Questions Descriptors

Introduction

1. Solving a system of equations using a table of values.

Graphs

1. Solving a system of equations using graphing.
2. Determine the number of solutions for a system of equations by analyzing the slopes of the lines.

Substitution

1. Solving a system of equations using the method of substitution.

Elimination

1. Solving a system of equations using the method of elimination.

Application

1. Solving a system of equations in an applied situation. Level of difficulty: 1.
2. Solving a system of equations in an applied situation. Level of difficulty: 2.
3. Solving a system of equations in an applied situation. Level of difficulty: 3.
4. Solving a system of equations in an applied situation. Level of difficulty: 4.
5. Solving a system of equations in an applied situation. Level of difficulty: 5.
6. Solving a system of equations in an applied situation. Level of difficulty: 6.

Factoring Quiz Questions Descriptors

Greatest Common Term

1. Factor a binomial of trinomial using the greatest common factor method. Level of difficulty: 1.
2. Factor a binomial of trinomial using the greatest common factor method. Level of difficulty: 2.
3. Factor a binomial of trinomial using the greatest common factor method. Level of difficulty: 3.
4. Factor a binomial of trinomial using the greatest common factor method. Level of difficulty: 4.
5. Factor a binomial of trinomial using the greatest common factor method. Level of difficulty: 5.
6. Factor a binomial of trinomial using the greatest common factor method. Level of difficulty: 6.

Difference of Squares

1. Factor a binomial using the difference of squares method. Level of difficulty: 1.
2. Factor a binomial using the difference of squares method. Level of difficulty: 2.
3. Factor a binomial using the difference of squares method. Level of difficulty: 3.
4. Factor a binomial using the difference of squares method. Level of difficulty: 4.
5. Factor a binomial using the difference of squares method. Level of difficulty: 5.
6. Factor a binomial using the difference of squares method. Level of difficulty: 6.

Perfect Squares

1. Factor a trinomial using the perfect squares method. Level of difficulty: 1.
2. Factor a trinomial using the perfect squares method. Level of difficulty: 2.
3. Factor a trinomial using the perfect squares method. Level of difficulty: 3.
4. Factor a trinomial using the perfect squares method. Level of difficulty: 4.
5. Factor a trinomial using the perfect squares method. Level of difficulty: 5.
6. Factor a trinomial using the perfect squares method. Level of difficulty: 6.

Simple Trinomials

1. Factor a trinomial using the simple trinomial method. Level of difficulty: 1.
2. Factor a trinomial using the simple trinomial method. Level of difficulty: 2.
3. Factor a trinomial using the simple trinomial method. Level of difficulty: 3.
4. Factor a trinomial using the simple trinomial method. Level of difficulty: 4.
5. Factor a trinomial using the simple trinomial method. Level of difficulty: 5.
6. Factor a trinomial using the simple trinomial method. Level of difficulty: 6.

Complex Trinomials

1. Factor a trinomial using the complex trinomial method. Level of difficulty: 1.
2. Factor a trinomial using the complex trinomial method. Level of difficulty: 2.

3. Factor a trinomial using the complex trinomial method. Level of difficulty: 3.
4. Factor a trinomial using the complex trinomial method. Level of difficulty: 4.
5. Factor a trinomial using the complex trinomial method. Level of difficulty: 5.
6. Factor a trinomial using the complex trinomial method. Level of difficulty: 6.

Multi-step

1. Completely factor a binomial.
2. Completely factor a trinomial.
3. Completely factor a trinomial.
4. Completely factor a trinomial.

Quadratic Equations Quiz Questions Descriptors

Introduction

1. Identify whether or not an equation is a quadratic equation.
2. Identify the root of a quadratic equation given a table of values. Level of difficulty: 1.
3. Identify the root of a quadratic equation given a table of values. Level of difficulty: 2.
4. Identify the root of a quadratic equation given a table of values. Level of difficulty: 3.

Isolation

1. Solve a quadratic equation using the method of isolation.

Factoring

1. Solve a quadratic equation using factoring. Level of difficulty: 1.
2. Solve a quadratic equation using factoring. Level of difficulty: 2.

Quadratic Formula

1. Solve a quadratic equation using the quadratic formula. Level of difficulty: 1.
2. Solve a quadratic equation using the quadratic formula. Level of difficulty: 2.
3. Evaluate the discriminant of a quadratic equation.

Quadratic Functions Quiz Questions Descriptors

Introduction

1. Given the graph of y = ax^2, determine the value of a.
2. Given the graph of y = ax^2 + c, determine the value of c.
3. Given the graph of y = ax^2 + c, determine the value of a and c.
4. Given the graph of y = (x - h)^2, determine the value of h.

Graphs

1. Determine realistic values for the domain of a quadratic function for a profit/loss situation.
2. Determine realistic values for the range of a quadratic function for a profit/loss situation.
3. Determine realistic values for the domain of a quadratic function that represents the height of a falling object as a function of time.
4. Place the graph of a quadratic function that represents the height of a falling object as a function of time.

5. Solve problems involving quadratic functions representing the height of a falling object as a function of time.
6. Determine the equation of a quadratic function that represents the height of a falling object as a function of time.

Curve of Best Fit

1. Determine the equation of a curve of best fit.
2. Extrapolate using a curve of best fit.

Inequalities Quiz Questions Descriptors

Introduction

1. Identify the correct and incorrect use of inequality signs.
2. Use inequality signs to compare two differing quantities.
3. Identify solutions of inequalities.

Solving Inequalities

1. Solve inequalities based on the equality $ax + b = c$.
2. Solve inequalities based on the equality $c = ax + b$.
3. Solve inequalities based on the equality $ax + b = cx + d$.
4. Solve inequalities based on the equality $a(bx + c) = dx + e$.
5. Solve inequalities based on the equality $a(bx + c) = d(ex + f)$.
6. Model and solve an inequality representing a real life situation (wedding).
7. Model and solve an inequality representing a real life situation (triangle).
8. Model and solve an inequality representing a real life situation (car rental).
9. Model and solve an inequality representing a real life situation (rectangle).

Linear Inequalities Quiz Questions Descriptors

Introduction

1. Graph a linear inequality.

Systems

1. Graph a system of linear inequalities.

Application

1. Solve a system of linear inequalities in an applied situation. Level of difficulty 1.
2. Solve a system of linear inequalities in an applied situation. Level of difficulty 2.
3. Solve a system of linear inequalities in an applied situation. Level of difficulty 3.
4. Solve a system of linear inequalities in an applied situation. Level of difficulty 4.

Functions Quiz Questions Descriptors

Introduction

1. Determine whether or not a given set of ordered pairs is a function.
2. Create a set of ordered pairs that is a function.
3. Determine whether or not a given table of values is a function.
4. Determine whether or not a given graph is a function.
5. Identify the dependent and independent variables from a verbal description of a function.
6. From a verbal description determine if a relation is a function, and if so, identify the independent variable.
7. Distinguish between correct and incorrect statements about a functional relationship, and its dependent and independent variables.
8. Match the verbal description of a function to its algebraic representation.

Domain

1. Identify the domain of a function represented by a set of ordered pairs.
2. Identify a function represented by a set of ordered pairs that corresponds to a given domain.
3. Identify the domain of a function represented by a table of values.
4. Identify the domain of a function represented by a table of values in a real life context.
5. Identify the domain of a function represented by a graph. Level of difficulty: 1.
6. Identify the domain of a function represented by a graph. Level of difficulty: 2.
7. From a verbal description express the domain of a function using set notation.
8. From an algebraic representation of a function determine what values must be excluded from the domain.
9. Identify the domain of a function from its algebraic representation.

Range

1. Identify the range of a function represented by a set of ordered pairs.
2. Identify a function represented by a set of ordered pairs that corresponds to a given range.
3. Identify the range of a function represented by a table of values.
4. Identify the range of a function represented by a table of values in a real life context.
5. Identify the range of a function represented by a graph. Level of difficulty: 1.
6. Identify the range of a function represented by a graph. Level of difficulty: 2.
7. From a verbal description express the range of a function using set notation.

Values and Zeros

1. Evaluate a function represented algebraically for a given value of the independent variable. Level of difficulty: 1.
2. Evaluate a function represented algebraically for a given value of the independent variable. Level of difficulty: 2.
3. Evaluate a function represented algebraically for a given value of the independent variable. Level of difficulty: 3.
4. Determine the zeros of a function from its algebraic representation. Level of difficulty: 1.
5. Determine the zeros of a function from its algebraic representation. Level of difficulty: 2.
6. Determine the zeros of a function from its graph.

Rational Expressions Quiz Questions Descriptors

Introduction

No questions.

Restrictions

1. Identify variable values for which a rational expression is undefined.

Simplifying

1. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 1.
2. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 2.
3. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 3.
4. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 4.
5. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 5.
6. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 6.
7. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 7.
8. Simplify a rational expression and identify any restrictions on the variables. Level of difficulty: 8.

Multiplying

1. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 1.
2. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 2.
3. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 3.
4. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 4.
5. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 5.
6. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 6.
7. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 7.
8. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 8.
9. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 9.
10. Multiply a rational expression and identify any restrictions on the variables. Level of difficulty: 10.

Dividing

1. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 1.
2. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 2.
3. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 3.
4. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 4.
5. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 5.
6. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 6.
7. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 7.
8. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 8.
9. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 9.
10. Divide a rational expression and identify any restrictions on the variables. Level of difficulty: 10.

Adding and Subtracting

1. Add and subtract rational expressions and identify any restrictions on the variables. Level of difficulty: 1.
2. Add and subtract rational expressions and identify any restrictions on the variables. Level of difficulty: 2.
3. Add and subtract rational expressions and identify any restrictions on the variables. Level of

difficulty: 3.

4. Add and subtract rational expressions and identify any restrictions on the variables. Level of difficulty: 4.
5. Add and subtract rational expressions and identify any restrictions on the variables. Level of difficulty: 5.
6. Add and subtract rational expressions and identify any restrictions on the variables. Level of difficulty: 6.

Additional Information and Activities

NUMBERS

Activity #1

Evaluate each expression using:

$$a = \frac{1}{3}$$

$$b = \frac{-1}{4}$$

$$c = 1\frac{1}{2}$$

1. $a + bc$
2. $2a - 4b + 2c$
3. $ab + c$
4. $c\,(a + b)$
5. $(a - c)\,(a + c)$
6. $-3\,(a - b + c)$
7. $\dfrac{c}{a+b}$
8. $\dfrac{a}{b} + \dfrac{b}{c}$

Activity #1 - Solutions

1.

$$= \frac{1}{3} + \left(\frac{-1}{4}\right)\left(1\frac{1}{2}\right)$$
$$= \frac{1}{3} + \left(\frac{-1}{4}\right)\left(\frac{3}{2}\right)$$
$$= \frac{1}{3} - \frac{3}{8}$$
$$= \frac{8}{24} - \frac{9}{24}$$
$$= \frac{-1}{24}$$

2.

$$\left(2 \times \frac{1}{3}\right) - \left(4 \times \frac{-1}{4}\right) + \left(2 \times \frac{3}{2}\right)$$
$$= \frac{2}{3} - (-1) + 3$$
$$= \frac{2}{3} + 1 + 3$$
$$= 4\frac{2}{3}$$

3.

$$\left(\frac{1}{3} \times \frac{-1}{4}\right) + 1\frac{1}{2}$$
$$= \frac{-1}{12} + \frac{18}{12}$$
$$= \frac{17}{12}$$
$$= 1\frac{5}{12}$$

4.

$$1\frac{1}{2}\left(\frac{1}{3} + \left(\frac{-1}{4}\right)\right)$$
$$= 1\frac{1}{2}\left(\frac{4}{12} - \frac{3}{12}\right)$$
$$= 1\frac{1}{2}\left(\frac{1}{12}\right)$$
$$= \frac{3}{2} \times \frac{1}{12}$$
$$= \frac{3}{24}$$
$$= \frac{1}{8}$$

5.

$$= \left(\frac{1}{3} - \frac{3}{2}\right)\left(\frac{1}{3} + \frac{3}{2}\right)$$

$$= \left(\frac{2}{6} - \frac{9}{6}\right)\left(\frac{2}{6} + \frac{9}{6}\right)$$

$$= \left(-\frac{7}{6}\right)\left(\frac{11}{6}\right)$$

$$= \frac{-77}{36}$$

$$= -2\frac{5}{36}$$

5. (alternate solution)

$$(a - c)(a + c) = a^2 - c^2$$

$$= \left(\frac{1}{3}\right)^2 - \left(1\frac{1}{2}\right)^2$$

$$= \left(\frac{1}{3}\right)^2 - \left(\frac{3}{2}\right)^2$$

$$= \frac{1}{9} - \frac{9}{4}$$

$$= \frac{4}{36} - \frac{81}{36}$$

$$= \frac{-77}{36}$$

$$= -2\frac{5}{36}$$

6.

$$-3\left(\frac{1}{3} - \left(\frac{-1}{4}\right) + 1\frac{1}{2}\right)$$

$$= -3\left(\frac{4}{12} + \frac{3}{12} + \frac{18}{12}\right)$$

$$= -3\left(\frac{25}{12}\right)$$

$$= \frac{-75}{12}$$

$$= -6\frac{1}{4}$$

7.

$$= \frac{1\frac{1}{2}}{\frac{1}{3}+\left(\frac{-1}{4}\right)}$$

$$= \frac{\frac{18}{12}}{\frac{4}{12}-\frac{3}{12}}$$

$$= \frac{18}{12} \div \frac{1}{12}$$

$$= \frac{18}{12} \times \frac{12}{1}$$

$$= 18$$

8.

$$= \frac{\frac{1}{3}}{\frac{-1}{4}} + \frac{\frac{-1}{4}}{1\frac{1}{2}}$$

$$= \left(\frac{1}{3} \div \left(\frac{-1}{4}\right)\right) + \left(\frac{-1}{4} \div \frac{3}{2}\right)$$

$$= \left(\frac{4}{12} \div \left(\frac{-3}{12}\right)\right) + \left(\frac{-1}{4} \div \frac{6}{4}\right)$$

$$= \left(\frac{4}{12} \times \frac{-12}{3}\right) + \left(\frac{-1}{4} \times \frac{4}{6}\right)$$

$$= \frac{-4}{3} + \left(\frac{-1}{6}\right)$$

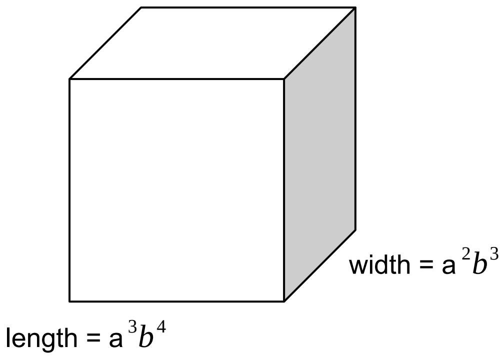
$$= -\left(\frac{8}{6}\right) - \left(\frac{1}{6}\right)$$

$$= \frac{-9}{6}$$

$$= -1\frac{1}{2}$$

Activity #2

1. What is an expression for the height of this box, given that it has a volume of a^9b^{11}?



2. Find the value of n in each equation.

(a) $(r^5)^8 = r^n$

(b) $(n^3)^3 = 512$

(c) $(3^6)^n = 3^{30}$

(d) $(p^7)^7 = p^n$

3. Express each product as a single power.

(a) $(p^2)^1 \times (p^6)^5$

(b) $(r^8)^6 \times (r^8)^1$

Activity #2 - Solutions

1.

$$v = l \times w \times h$$

$$h = \frac{v}{l \times w}$$

$$= \left(a^9 b^{11}\right) \div \left(a^3 b^4 \times a^2 b^3\right)$$

$$= \frac{\left(a^9 b^{11}\right)}{\left(a^5 b^7\right)}$$

$$= a^4 b^4$$

2. (a)

$$\left(r^5\right)^8 = r^n$$

$$r^{40} = r^n$$

$$40 = n$$

(b)

$$\left(n^3\right)^3 = 512$$

$$n^9 = 512$$

$$n^9 = 2^9$$

$$n = 2$$

(c)

$$\left(3^6\right)^n = 3^{30}$$

$$3^{6n} = 3^{30}$$

$$6n = 30$$

$$n = 5$$

(d)

$$\left(p^7\right)^7 = p^n$$

$$p^{49} = p^n$$

$$49 = n$$

3. (a)

$$\left(p^2\right)^1 \times \left(p^6\right)^5$$

$$= p^2 \times p^{30}$$

$$= p^{32}$$

(b)

$$\left(r^8\right)^6 \times \left(r^8\right)^1$$

$$= r^{48} \times r^8$$

$$= r^{56}$$

Activity #3

A UFO travels at 1.5538×10^4 km/h. An airplane travels at 2173 km/h. How many times faster is the UFO than the airplane? Round the answer to the nearest whole number.

Activity #3 – Solution

Ratio

$$= \frac{\left(1.5538\times10^4\right)}{2173}$$

$$= \frac{\left(1.5538\times10^4\right)}{\left(2.173\times10^3\right)}$$

$$= \frac{(1.5538)}{(2.173)}\times\frac{\left(10^4\right)}{\left(10^3\right)}$$

$$\approx 0.7150\times10^1$$

$$\approx 7.150$$

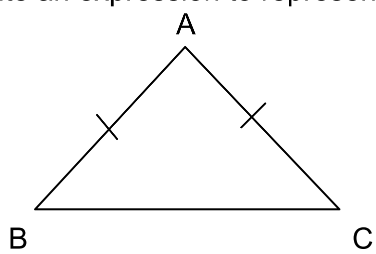
The UFO is approximately 7 times faster than the airplane.

POLYNOMIALS

ACTIVITY #1

1.

(a) Write an expression to represent the perimetre of this isosceles triangle.



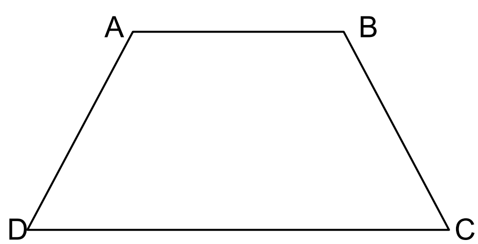
$|AB| = 2x^2 + 2x - 19$

$|BC| = -3x^2 + 3x + 23$

(b) Find the perimetre when x = 3 cm.

2.

(a) Write an expression to represent the length of side AD if the perimetre of this trapezoid is 10a – 11b – 2c + 122.



|AB| = 5a – b – c + 60
|BC| = 3a – 2b + 2c + 20
|DC| = a – 3b – 4c + 45

(b) What is the length of AD if a = -8, b = -2, c = 6?

ACTIVITY #2

You are decorating 300 cookies and require these supplies.

Item	Cost
icing sugar	8s
sprinkles	s – 2
food colouring	s – 1
gum drops	2s
flavouring	s
jelly beans	2s

(a) Write and simplify an expression that represents the total cost of supplies.

(b) If the flavouring costs $4, what is the total cost of supplies?

(c) You have $50 to spend. What could you eliminate to come within budget and still be able to decorate the cookies?

ACTIVITY #3

1. What is the area of a rectangle with these dimensions?

Length = $-6x^4y^7z^4$
Width = $3x^9y^2z^2$

2. Expand and simplify this expression. $-4x^2\left(6x^2+8y^8\right)$

3. What is the expanded form of $\left(-3x^7\right)\left(7xy^5+3x^4y^9\right)$?

4. What is the expanded form of $7x^8\left(7x^9y^4+4z^9\right)$?

5. What is the expanded form of $-6x^7y^6\left(-7x^5y^9+6x^7y^4+9x^4+y\right)$?

6. What is the expanded form of $-6x\left(2x-4y\right)-x\left(-x+2y\right)$?

7. What is the expanded form of $x^2\left(-3x^2-5x+4\right)+7x^2\left(6x^2-3x+5\right)$?

ACTIVITY #1 - Solutions

1. (a) P = AB + BC + AC

= $\left(2x^2 + 2x - 19\right) + \left(-3x^2 + 3x + 23\right) + \left(2x^2 + 2x - 19\right)$

= $2x^2 + 2x - 19 - 3x^2 + 3x + 23 + 2x^2 + 2x - 19$

= $x^2 + 7x - 15$

(b) Substitution for x

= $3^2 + 7\left(3\right) - 15$

= $9 + 21 - 15$

= 15 cm

2. (a) P = AB + BC + CD + AD

AD = P – (AB + BC + CD)

= (10a – 11b – 2c + 122) – ((5a – b – c + 60) + (3a – 2b + 2c + 20) + (a – 3b – 4c + 45))

= 10a – 11b – 2c + 122 – (9a – 6b – 3c + 125)

= 10a – 11b – 2c + 122 – 9a + 6b + 3c – 125

= a – 5b + c - 3

(b) Substitution to find length of AD.

AD = a – 5b + c – 3

= (-8) – 5 (-2) + (6) – 3

= - 8 + 10 + 6 – 3

= 5

ACTIVITY #2 – Solutions

(a) Cost = 8s + (s – 2) + (s – 1) + 2s + s + 2s

= 8s + s – 2 + s – 1 + 2s + s + 2s

= 15s – 3

(b) Cost = 15s - 3

= 15 x 4 - 3

= 60 - 3

= $57

(c) You need to reduce your expenses by $7. The cost of the gum drops is $8 and the cost of the jelly beans is $8. If you eliminate either of these you will be within budget.

ACTIVITY #3 – Solutions

1. $-18x^{13}y^{9}z^{6}$

2. $-24x^{4}-32x^{2}y^{8}$

3. $-21x^{8}y^{5}-9x^{11}y^{9}$

4. $49x^{17}y^{4}+28x^{8}z^{9}$

5. $42x^{12}y^{15}-36x^{14}y^{10}-54x^{11}y^{6}-6x^{7}y^{7}$

6. $-11x^{2}+22xy$

7. $39x^{4}-26x^{3}+39x^{2}$

EQUATIONS

Methods leading to solutions of systems of equations are required to solve problems which arise from real-life situations. Two such cases are cited.

Case 1

Nancy, who is 47, wondered just how old her aunt Edith was. Edith refused to tell anyone her age, but has been a senior citizen for some time. After much pressure from Nancy, Edith admitted that some years ago, she was exactly three times as old as Nancy, and nine years before that, was five times as old. “Aha!” said Nancy. “Now I know how old she is, but I had to do some figuring.”

Case 2

Mr. Smith had driven into the city from his new country home to buy some 25 cm square tiles. He wanted to tile the entire floor of his basement, but he forgot the dimensions. However, he did remember that the perimetre of the basement was 32 m and that the basement was 4 m longer than it was wide. Therefore, he was able to buy the correct number of tiles. What was that number?

In both cases, the situation has to be converted into algebraic statements of two unknowns, and each system must be solved. This section deals with techniques used to solve such systems of equations.

Two features need development. First, the student must develop facility in converting words into algebraic statements with two variables. Second, the student must learn how to solve systems of equations involving two equations with two variables. Activity sheets designed to assist in the development of these phases are included.

Example I

TOPIC: Solving Systems of Equations Using Tables of Values.

This method requires the equations to be in the "slope-intercept form". If the equations are not in this form in practice exercises, they must be converted to it.

Solve by using tables of values.

$$\begin{cases} y = 4x - 7 \\ y = -x + 8 \end{cases}$$

Prepare a table with headings for *x* , and the two *y*-values. Evaluate both *y*-values for a domain of *x* . (e.g. $-2 \leq x \leq 6$)

x	$4x-7$	$-x+8$
-2	-8 - 7 = -15	2 + 8 = 10
-1	-4 - 7 = -11	1 + 8 = 9
0	0 - 7 = -7	0 + 8 = 8
1	4 - 7 = -3	-1 + 8 = 7
2	8 - 7 = 1	-2 + 8 = 6
3	12 - 7 = 5	-3 + 8 = 5
4	16 - 7 = 9	-4 + 8 = 4
5	20 - 7 = 13	-5 + 8 = 3
6	24 - 7 = 17	-6 + 8 = 2

Note that when *x* = 3, both *y*-values are 5. Therefore, the solution is *x* = 3, *y* = 5.

Note that as *x* increases in value from -2, the difference in the two *y*-values decreases progressively from 25 to 20 to 15 to 10 to 5 to 0 at *x* = 3. The objective is to find a value of *x* such that there is no difference between the *y*-values.

Example II

TOPIC: Solving Systems of Equations Using Tables of Values.

Solve By using tables of values

$$\begin{cases} y = 5x + 16 \\ y = -2x - 5 \end{cases}$$

x	$5x+16$	$-2x-5$
-4	-4	3
-3	1	1
-2	It is not necessary to continue once the solution has been found.	
-1		
0		
1		

Note that when $x = -3$, both *y*-values are 1 . Therefore, the solution is $x = -3, y = 1$.

ACTIVITY #1

1. Identify which lettered line on the graph matches the given equation. Fill in the number of the line after each equation..

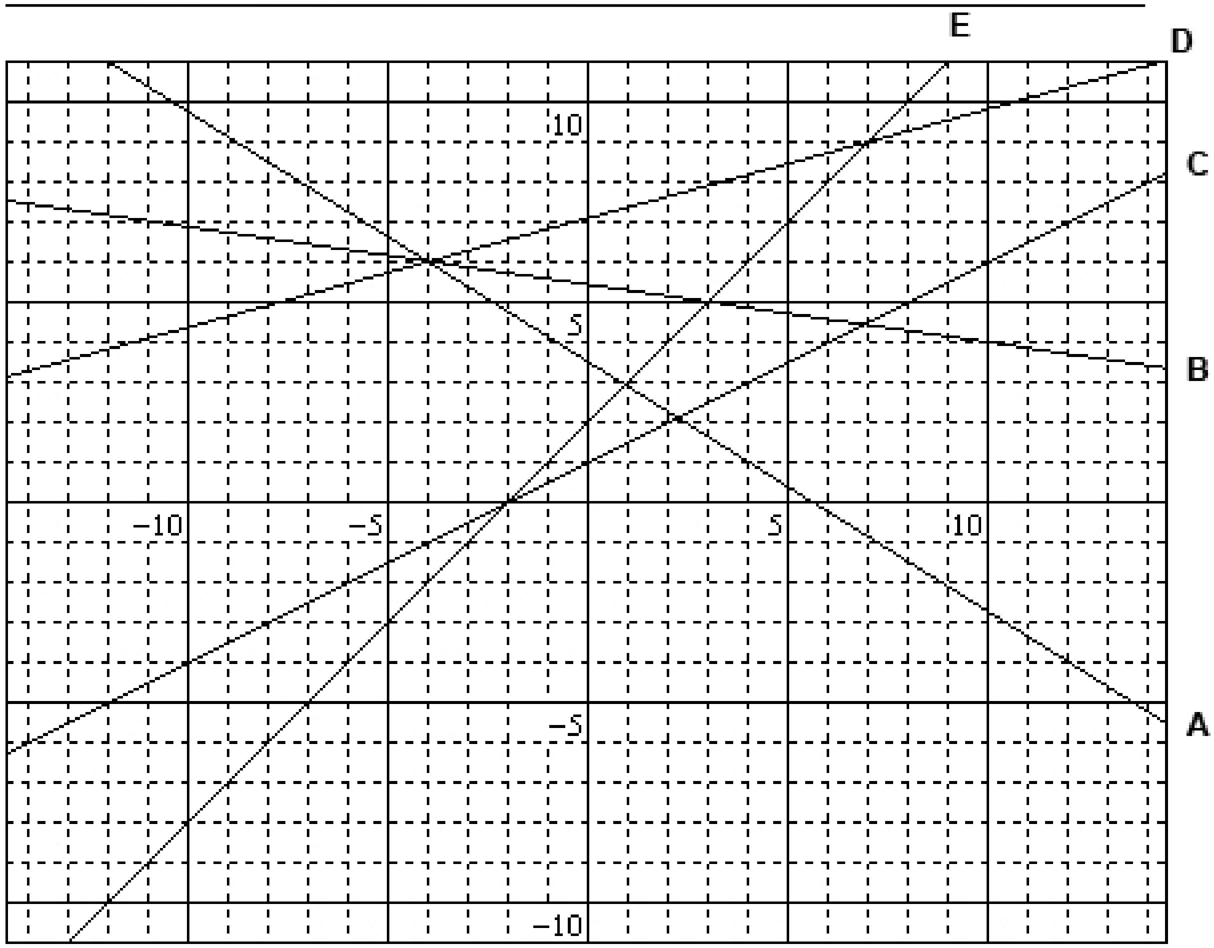
$x - 2y = 2$ __________

$3x - 11y = -78$ __________

$x + 7y - 38 = 0$ __________

$5x + 8y - 28 = 0$ __________

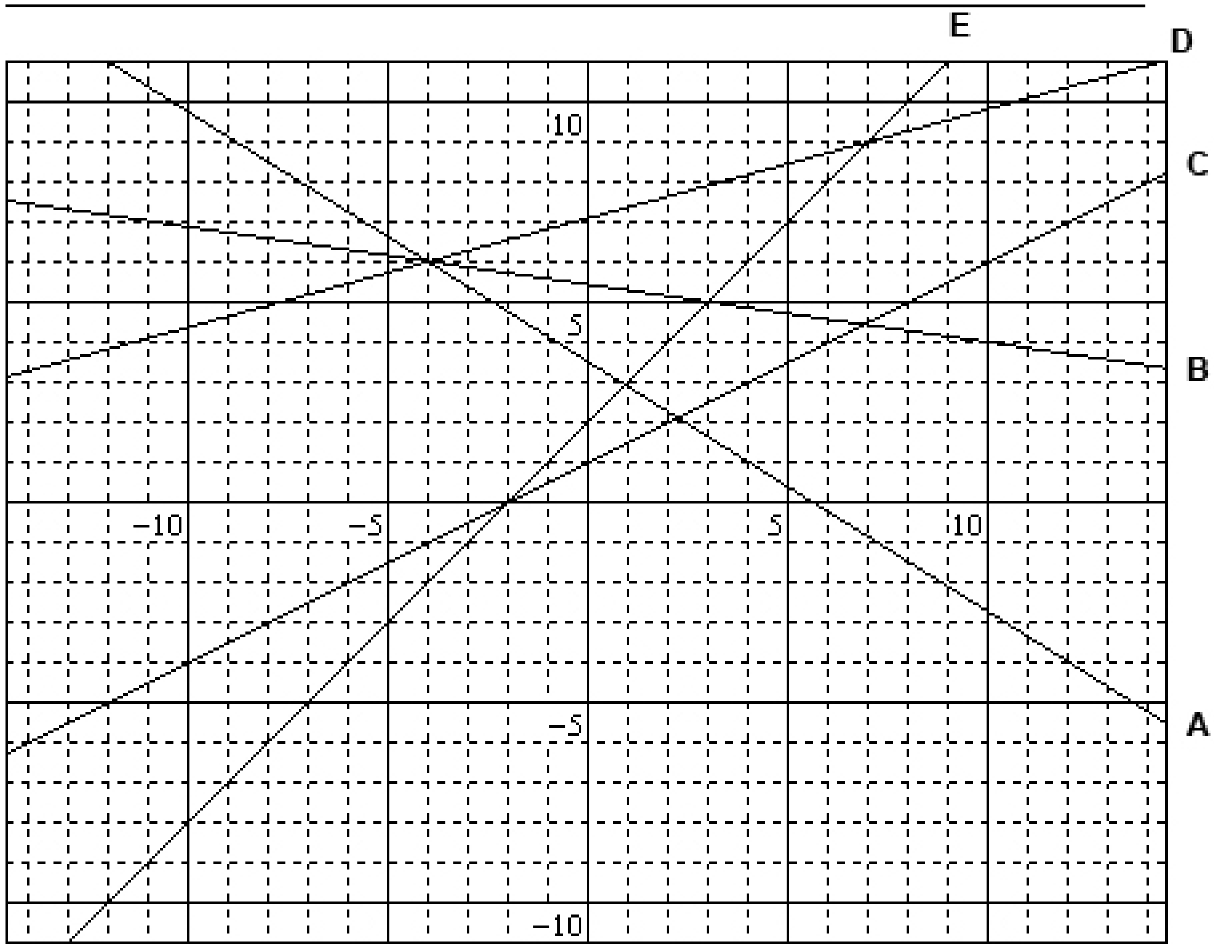
$x - y + 2 = 0$ __________



ACTIVITY #1 - Solutions

1. Identify which lettered line on the graph matches the given equation. Fill in the number of the line after each equation..

$x-2y=2$	**C**
$3x-11y=-78$	**D**
$x+7y-38=0$	**B**
$5x+8y-28=0$	**A**
$x-y+2=0$	**E**



ACTIVITY #2

1. Using Activity Sheet I (information), solve the following systems of equations.

a) $\begin{cases} 5x + 8y - 28 = 0 \\ x - 2y = 2 \end{cases}$ Answer $x =$ ☐ $y =$ ☐

b) $\begin{cases} 3x - 11y + 78 = 0 \\ x + 7y = 38 \end{cases}$ Answer $x =$ ☐ $y =$ ☐

c) $\begin{cases} x - y + 2 = 0 \\ x + 7y = 38 \end{cases}$ Answer $x =$ ☐ $y =$ ☐

d) $\begin{cases} x - 2y = 2 \\ x - y + 2 = 0 \end{cases}$ Answer $x =$ ☐ $y =$ ☐

e) $\begin{cases} x + 7y - 38 = 0 \\ x - 2y = 2 \end{cases}$ Answer $x =$ ☐ $y =$ ☐

2. a) How many pairs of equations have solutions which can be read accurately from the graph? ☐

   b) How many solutions have to be guessed at? ☐

   c) How many solutions cannot be found on this graph? ☐

3. What would be a good guess for the solution of the system formed by line A and line E?

   $x =$ ☐ $y =$ ☐

4. Describe what you could do to find the solution to the system of line C and line D.

ACTIVITY #2 - Solutions

1. Using Activity Sheet I (information), solve the following systems of equations.

a) $\begin{cases} 5x + 8y - 28 = 0 \\ x - 2y = 2 \end{cases}$ Answer $x =$ **4** $y =$ **1**

b) $\begin{cases} 3x - 11y + 78 = 0 \\ x + 7y = 38 \end{cases}$ Answer $x =$ **-4** $y =$ **6**

c) $\begin{cases} x - y + 2 = 0 \\ x + 7y = 38 \end{cases}$ Answer $x =$ **3** $y =$ **5**

d) $\begin{cases} x - 2y = 2 \\ x - y + 2 = 0 \end{cases}$ Answer $x =$ **-6** $y =$ **-4**

e) $\begin{cases} x + 7y - 38 = 0 \\ x - 2y = 2 \end{cases}$ Answer $x =$ **10** $y =$ **4**

2. a) How many pairs of equations have solutions which can be read accurately from the graph? **8**

   b) How many solutions have to be guessed at? **1**

   c) How many solutions cannot be found on this graph? **1**

3. What would be a good guess for the solution of the system formed by line A and line E?

   $x =$ **1.9** $y =$ **3.9**

4. Describe what you could do to find the solution to the system of line C and line D.

   I would draw their graphs using a different scale.

ACTIVITY #3

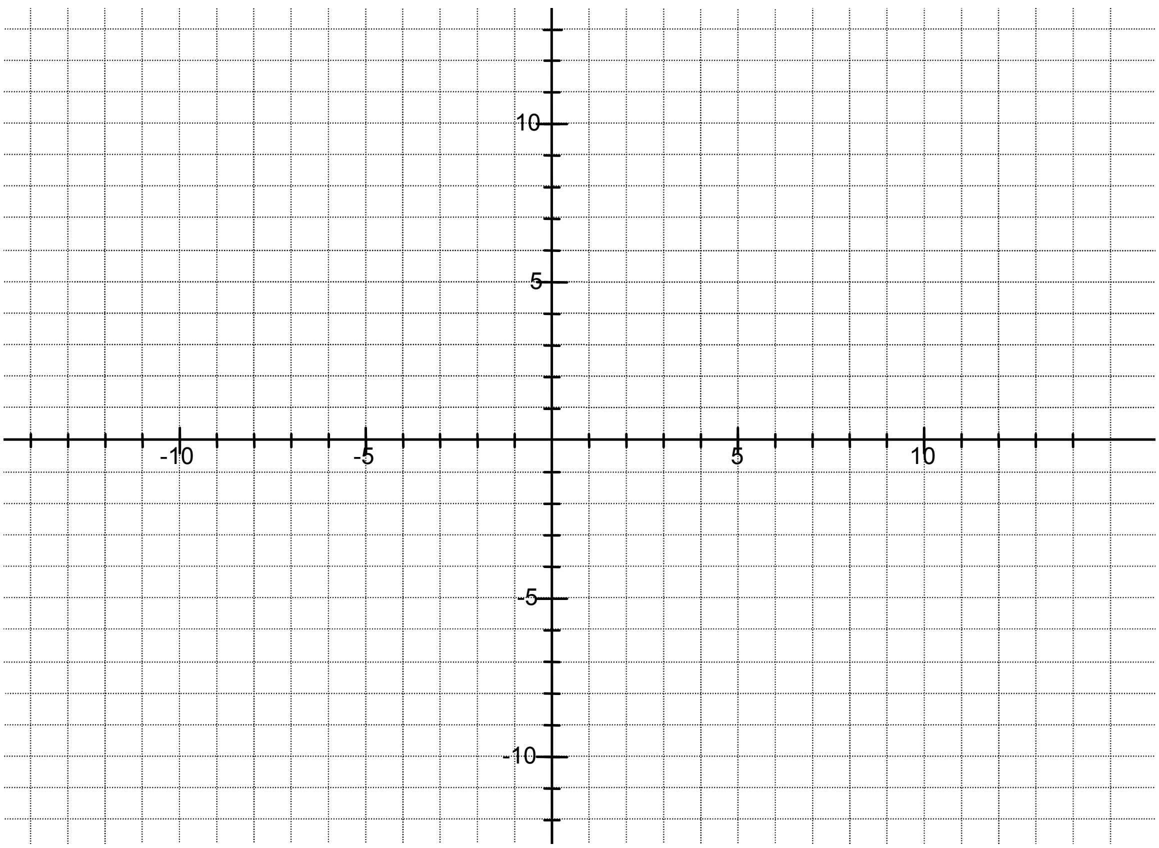
1. The lines forming the three sides of a triangle can be described by the following three equations.

A: $11x + y = 29$

B: $2x + 3y = -6$

C: $5x - 8y + 46 = 0$

Graph the three lines on the grid below and determine the coordinates of the vertices of the triangle.



The vertices of the triangle are (,) , (,) , (,) .

ACTIVITY #3 - Solutions

1. The lines forming the three sides of a triangle can be described by the following three equations.

A: $11x + y = 29$

B: $2x + 3y = -6$

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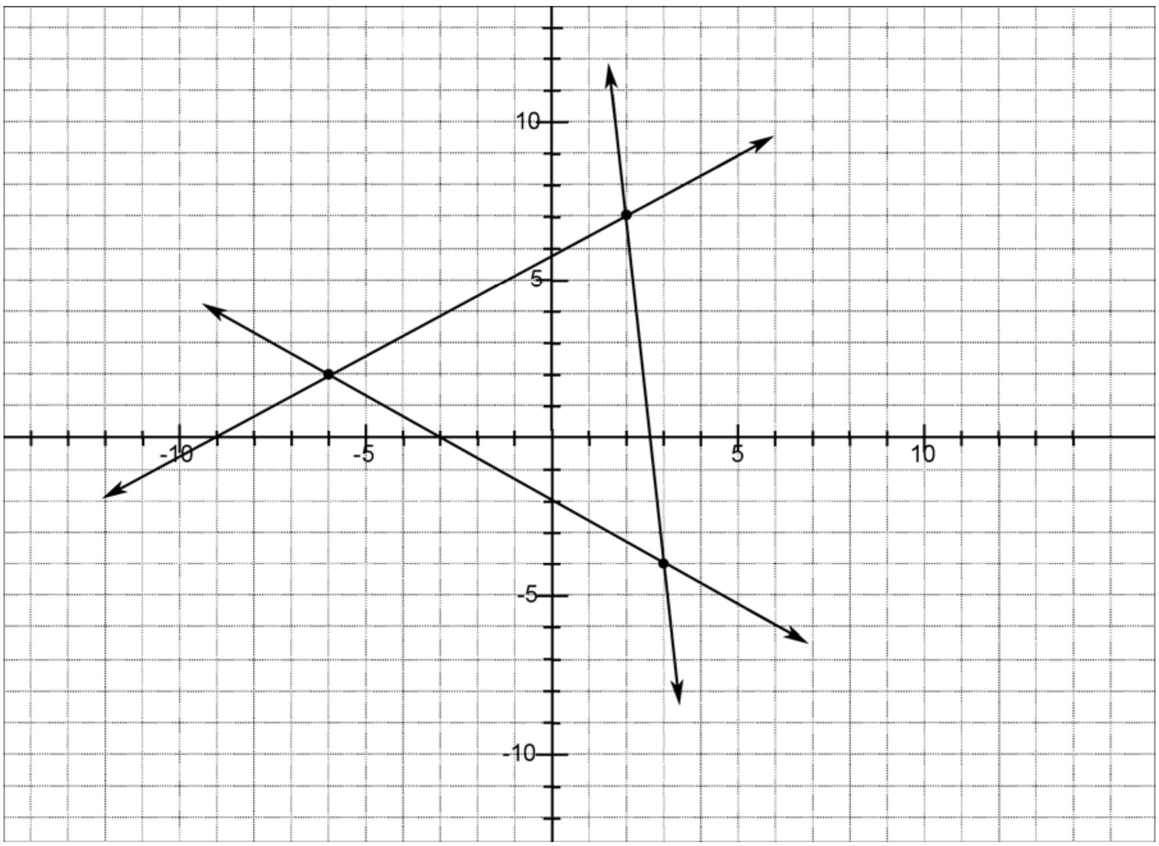
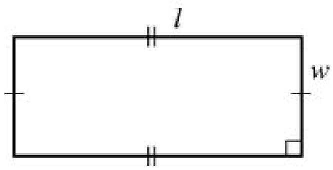
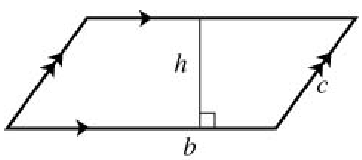
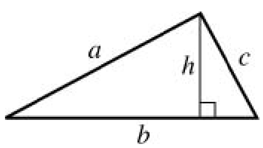
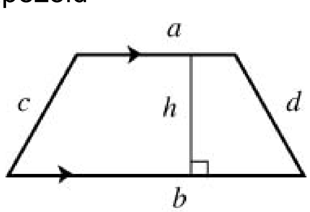
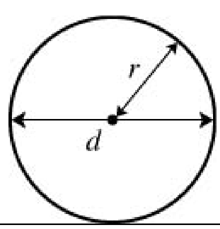
Graph the three lines on the grid below and determine the coordinates of the vertices of the triangle.

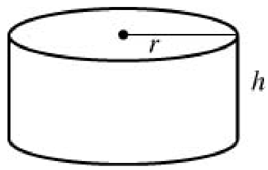
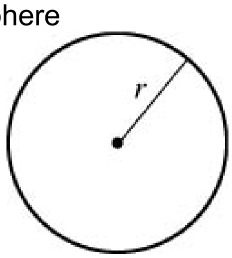
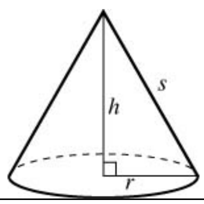
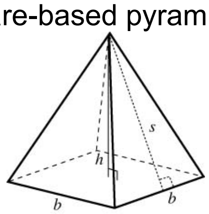
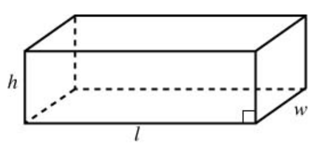
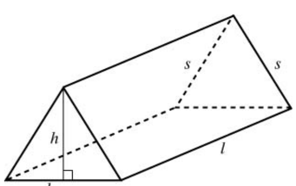


The vertices of the triangle are (-6, 2), (2, 7), (3,-4) .

GEOMETRIC RELATIONSHIPS
ACTIVITY #1

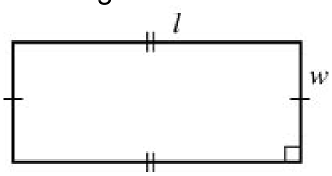
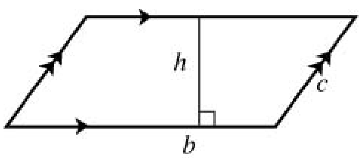
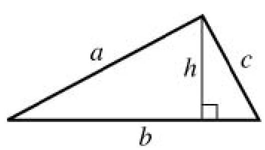
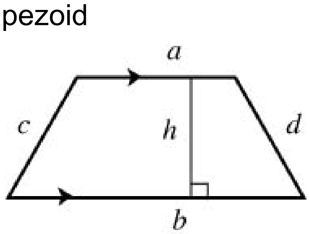
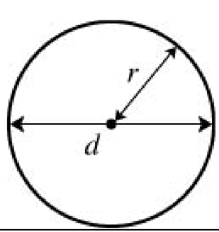
Complete the chart by filling in the boxes with formulas.

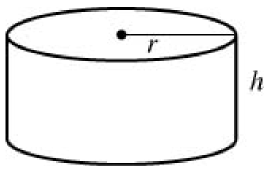
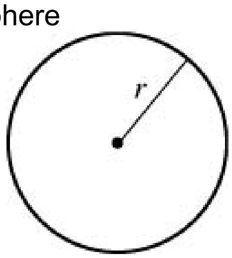
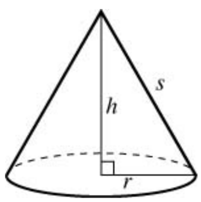
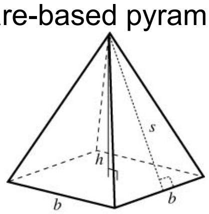
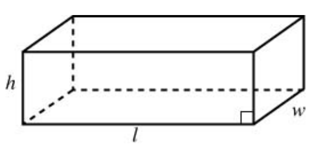
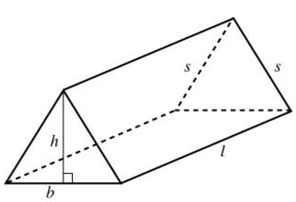
Geometric Figure	Perimetre	Area/Surface Area
Rectangle 		
Parallelogram 		
Triangle 		
Trapezoid 		
Circle 		

Geometric Figure	Area/Surface Area	Volume
Cylinder 		
Sphere 		
Cone 		
Square-based pyramid 		
Rectangular prism 		
Isosceles triangular prism 		

GEOMETRIC RELATIONSHIPS
ACTIVITY #1 – Solutions

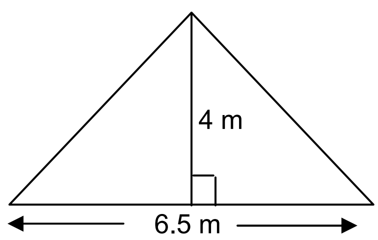
Complete the chart by filling in the boxes with formulas.

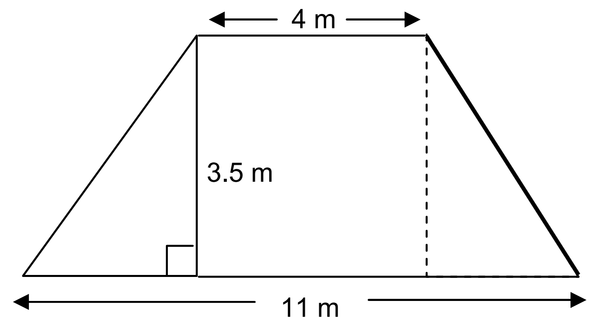
Geometric Figure	**Perimetre**	**Area/Surface Area**
Rectangle 	*P = 2l + 2w* *P = 2(l + w)*	*A = lw*
Parallelogram 	*P = b + b + c + c* *P = 2b + 2c*	*A = bh*
Triangle 	*P = a + b + c*	$A = \frac{bh}{2}$ **or** $A = \frac{1}{2}\ bh$
Trapezoid 	*P = a + b + c + d*	$A = \frac{(a+b)h}{2}$ **or** $A = \frac{1}{2}\ (a + b)h$
Circle 	$C = \pi d$ **or** $C = 2\pi r$	$A = \pi r^2$

Geometric Figure	Area/Surface Area	Volume
Cylinder 	$A_{top} = \pi r^2$ $A_{base} = \pi r^2$ $A_{side} = 2\pi rh$ $A_{total} = \pi r^2 + \pi rh$	$V = \pi r^2 h$
Sphere 	$A = \pi r^2$	$v = \frac{4}{3}\pi^3$
Cone 	$A_{cone} = \pi rs$ $A_{base} = \pi r^2$ $A_{total} = A_{cone} + A_{base}$	$v = \frac{1}{3}\pi^2 h$
Square-based pyramid 	$A_{triangle} = \frac{1}{2}bs$ (for each triangle) $A_{base} = b^2$ $A_{total} = A_{triangles} + A_{base}$	$V = \frac{1}{3}b^2 h$
Rectangular prism 	$A_{total} = wh + wh + lw + lw + lh + lh$ $A = 2(wh + lw + lh)$	$V = lwh$
Isosceles triangular prism 	$A_{triangle} = \frac{1}{2}bh$(for each triangle) $A_{rectangles} = ls + lb + ls$ $A_{total} = A_{rectangles} + A_{triangles}$	$V = \frac{1}{2}bhl$

ACTIVITY #2

You are about to re-shingle the roof of your house. The roof has two triangular sections and two trapezoidal sections.





1. Calculate the total surface area.

2. Add 5% of the area to be purchased to allow for waste shingling.

3. If one bundle of shingles covers 3 m^2, how many bundles are required?

ACTIVITY #2 – Solutions

1. Area of triangles

$A = \frac{bh}{2}$

$A = \frac{6.5 \times 4}{2}$

$= 13\ m^2$

$A\ Two\Delta = 2 \times 13\ m^2$

$= 26\ m^2$

Area of trapezoids

$A = \frac{(a+b)h}{2}$

$= \frac{(4+11)3.5}{2}$

$= 26.25\ m^2$

$A\ Two\Delta = 2 \times 26.25\ m^2$

$= 52.5\ m^2$

Total area $26 + 52.5 = 78.5\ m^2$

2. Plus 5% waste

$78.5\ m^2$ x $5\% \approx 3.9\ m^2$

Total area

$78.5 + 3.9 = 82.4\ m^2$

3. Number of bundles

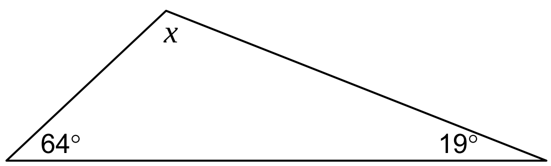
$82.4\ m^2 \div 3 \approx 27.47$

Therefore, 28 bundles are required.

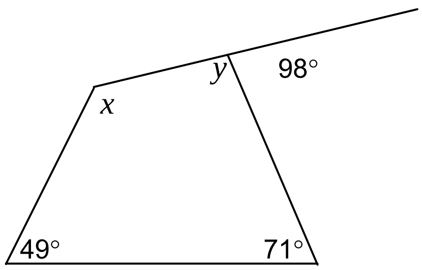
ACTIVITY #3

Find the measure of each angle.

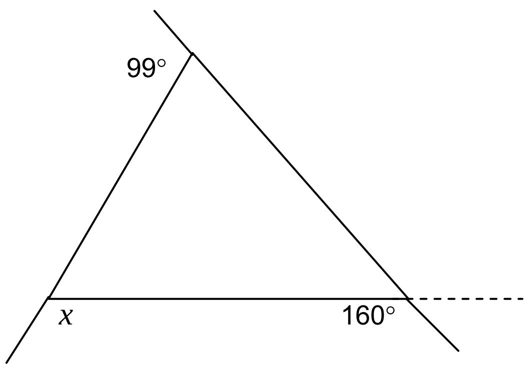
(a)



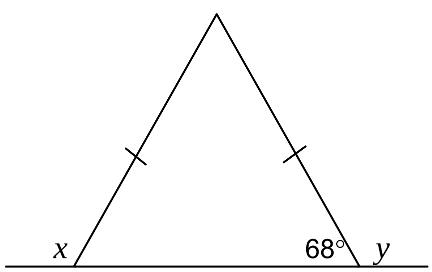
(b)



(c)



(d)



ACTIVITY #3 – Solutions

(a) $x + 64° + 19° = 180°$

$x = 180° - 64° - 19°$

$x = 97°$

(b) $y + 98° = 180°$

$y = 82°$

$x + y + 49° + 71° = 360°$

$x + 202° = 360°$

$x = 158°$

(c) $x + 99° + 160° = 360°$

$x + 259° = 360°$

$x = 101°$

(d) $x = y = 180° - 68°$

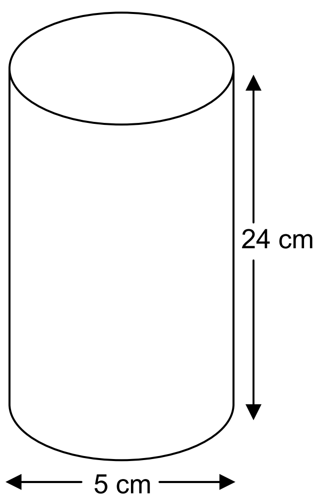
$x = y = 112°$

ACTIVITY #4

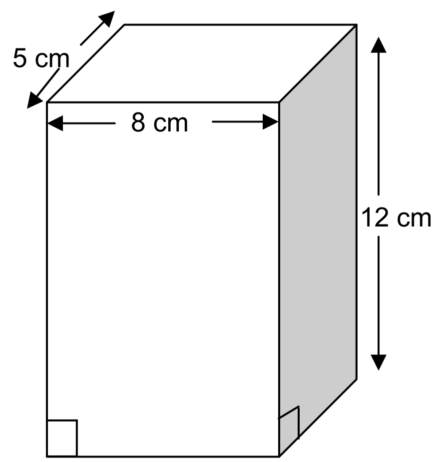
You are to select the container that is the most cost effective to produce, related to the amount of liquid it can hold. Explain your decision.

The cost of materials is \$0.22 per cm^2.

A.



B.



ACTIVITY #4 – Solutions

1. I determined the volume of each container.

A. $V = \pi r^2 h$
$\approx 3.14 \times 6.25 \times 24$
$\approx 471\ cm^3$

B. $V = 5 \times 8 \times 12$
$= 480\ cm^3$

2. I determined the surface area of each container.

A. SA = $2\pi rh + 2\pi r^2$

SA $\approx (2\times 3.14\times 2.5\times 24)+(2\times 3.14\times 6.25)$
$\approx 376.8 – 39.25$
$\approx 416.1\ cm^2$

B. SA $= 2\ lh + 2\ wh + 1\ lw$

SA $= (2 \times 8 \times 12) + (2 \times 5 \times 12) + (2 \times 8 \times 5)$
$= 192 + 120 + 80$
$= 392\ cm^2$

3. Cost of materials.

A. $416.1\ cm^2 \times \$0.22$
= $91.54

B. $391\ cm^2 \times \$0.22$
= $86.02

Conclusion:

Container A costs $91.54 to hold 471 cm^3 or 5.14 cm^3/dollar.

Container B costs $86.02 to hold 480 cm^3 or 5.58 cm^3/dollar.

Therefore, Container A is more cost effective.

STATISTICAL RELATIONSHIPS

ACTIVITY #1

TOPICS: Collecting Data, Organizing and Representing Data through Tables, Charts and Graphs, Measures of Central Tendency.

1. From a safe observation point, record the number of people in each of 50 passing automobiles.
2. Organize the results in a table.
3. Represent the data using a variety of appropriate charts and graphs.
4. Calculate the mean, median and mode for the data set.
5. Repeat the experiment at different times of the day and compare the results.

ACTIVITY #2

TOPICS: Organizing and Representing Data through Tables, Charts and Graphs.

A medical laboratory types the blood of 40 people with the following results:

A	O	B	O	O	AB	A	B	B	O
B	O	O	AB	O	A	O	B	A	AB
O	A	B	O	A	O	A	O	O	B
A	O	B	O	O	A	O	O	AB	A

1. Organize these results into a table.

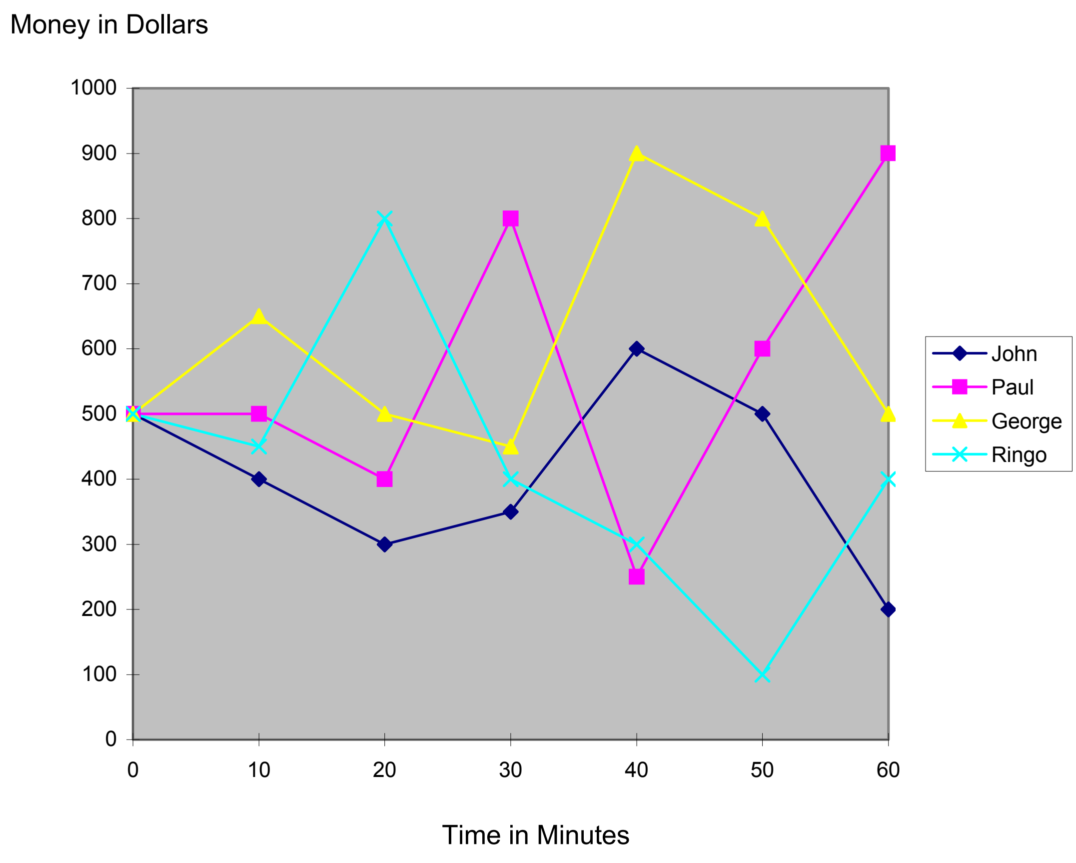
2. Represent this set of data using a variety of appropriate charts and graphs.

ACTIVITY #3

TOPICS: Interpreting Graphical Representations of Data.

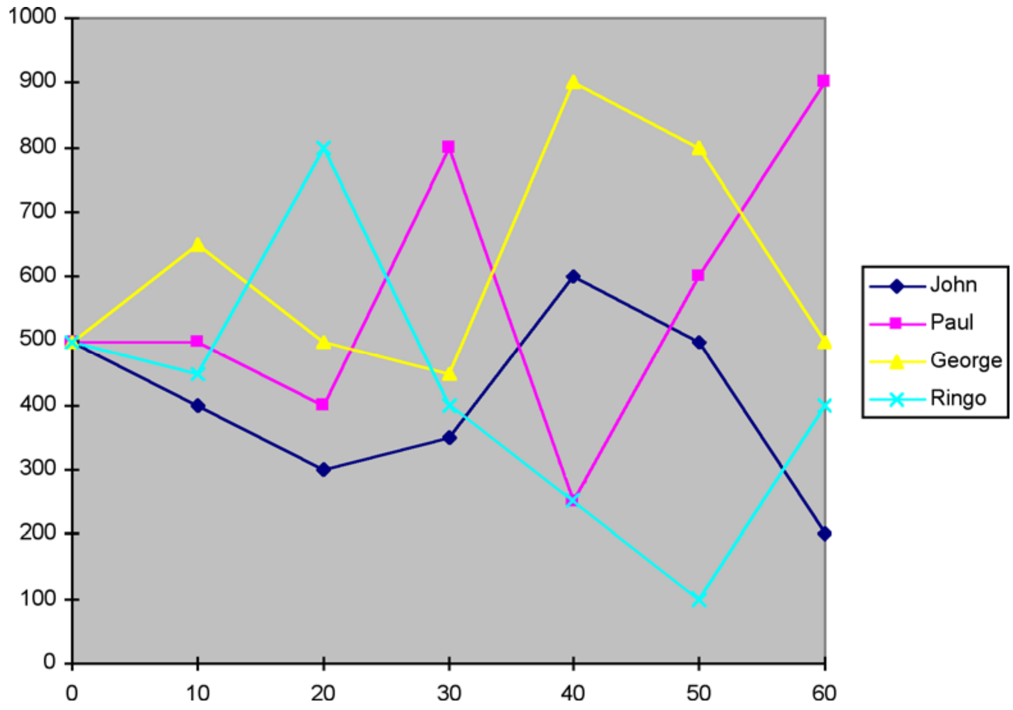
John, Paul, George and Ringo play a game in which each player begins with $500 in play money. During the game, money is won and lost and the player with the most money at the end of 60 minutes is the winner. The graph provided shows the amount of money held by each player (on the horizontal axis) in ten minute intervals (on the vertical axis). Using this graph, answer the following questions:

1. How much money did George have after 30 minutes?
2. Who was in last place after the first 20 minutes?
3. Who was in the lead at the end of more of the 10 minute intervals than any other player?
4. At which 10 minute interval did Paul and Ringo have the same amount of money? How much money did each have?
5. Was there a player who was never in the lead at the end of any 10 minute interval? If so, who was it?
6. Which player had the largest win/loss in a single 10 minute interval? How much money was involved?
7. Who won the game and with how much money?
8. Compose a play-by-play commentary describing the game.



ACTIVITY # 3 - Solutions

1. $450
2. John ($300)
3. George (3 times)
4. 40 minutes $250
5. Yes/John
6. Paul with a loss of $550
7. Paul $900



LINEAR FUNCTIONS

ACTIVITY #1

1. Using a strip of paper or a measuring tape, measure the diametre and circumference of five round objects.

2. Record the measurements in a table.

3. Graph the data. Place the diametres on the x-axis and the circumferences on the y-axis.

4. Fit the data with a straight line. How well does the line fit the data?

5. Determine the slope of the line.

6. Use the data and graph to make conjectures about the relationship between diametre and circumference.

ACTIVITY #2

A hot air balloon is released from the top of a 150 m building and rises at a rate of 7.5 m/sec. Create a graph showing the rise of the balloon for the first 30 seconds.

ACTIVITY #3

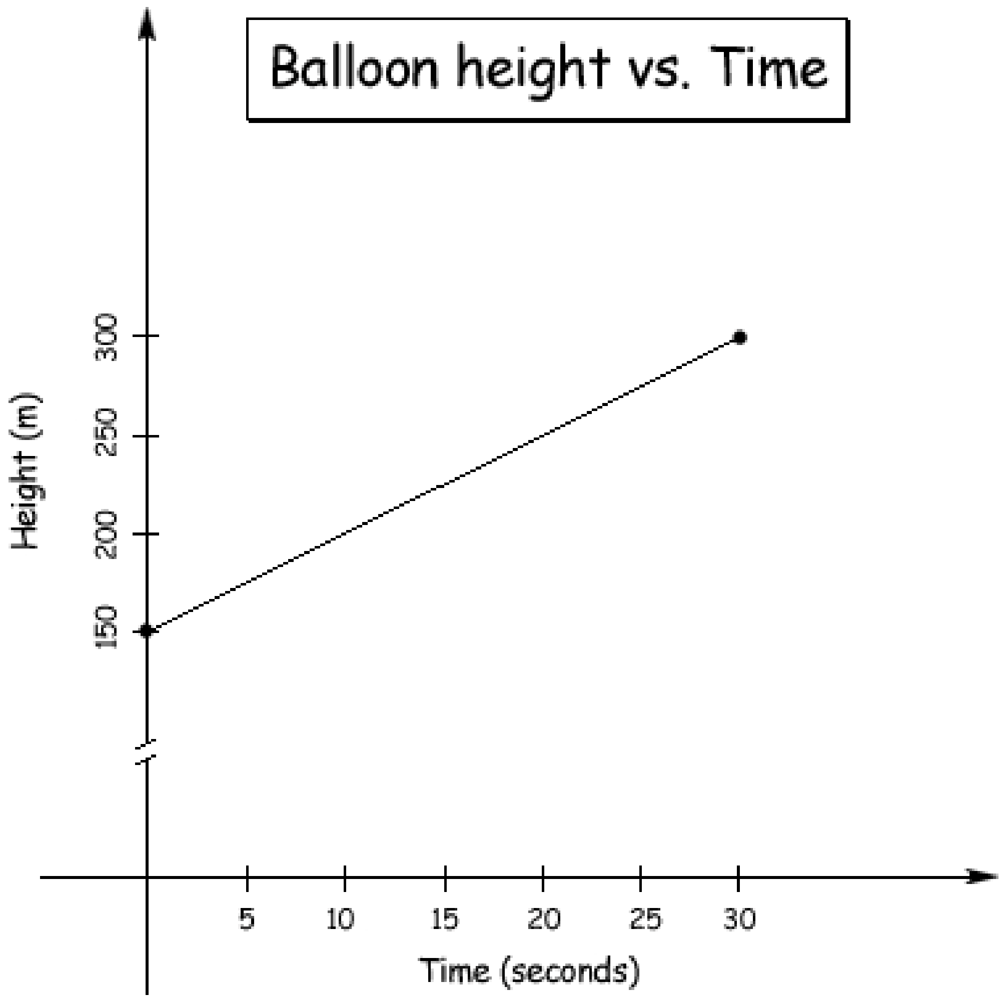
You are booking a caterer for a party. The cost of the meal depends on the number of guests served plus a flat fee.

The cost for serving 25 people is $165 and the cost for 40 guests is $225.

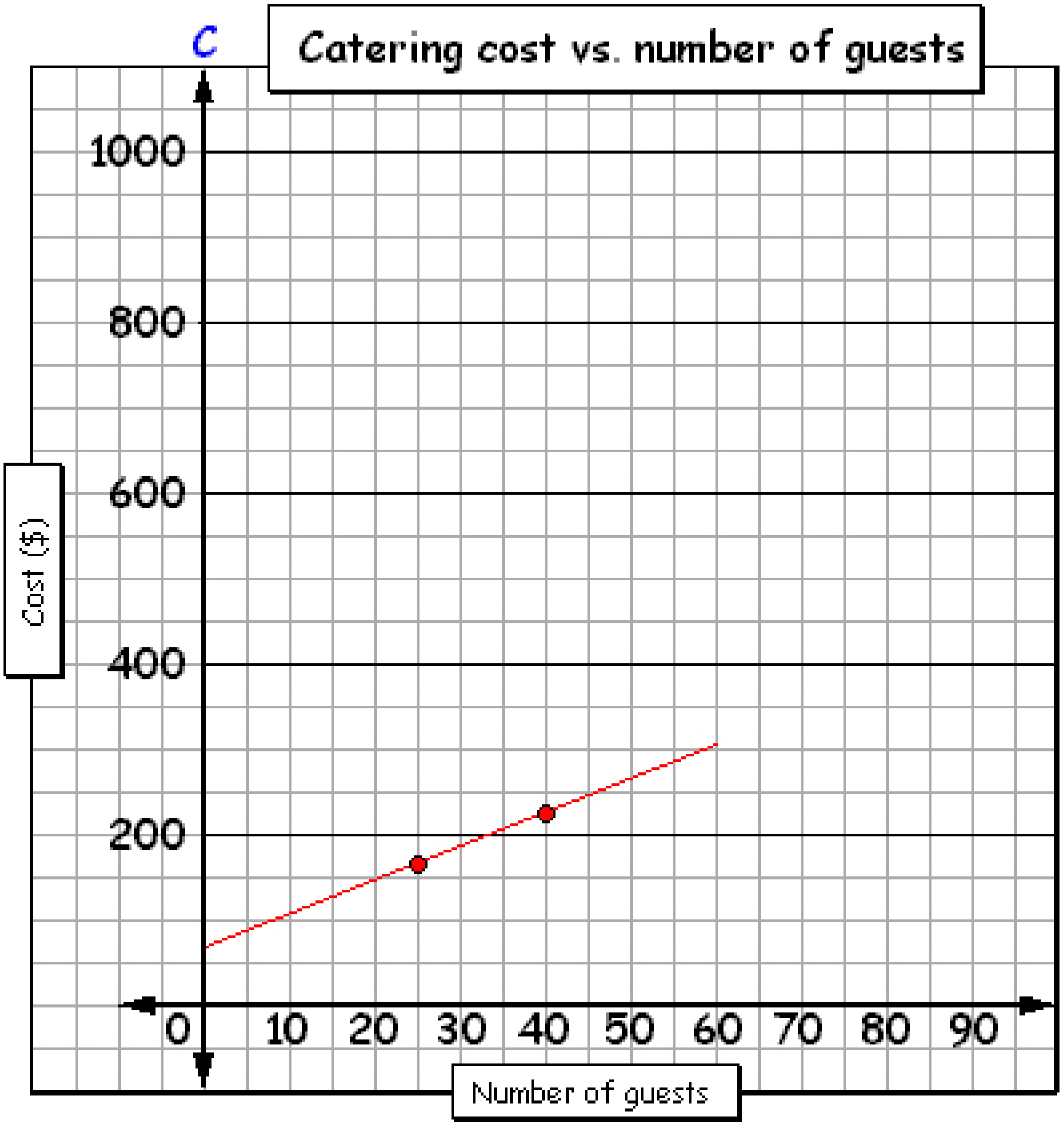
(a) Plot this information on a graph.

(b) Analyze the graph. What is the rental cost for the hall?

ACTIVITY # 2 – Solutions



ACTIVITY # 3 – Solutions



(b) $65.00, shown by the C-intercept.

LINEAR SYSTEMS

ACTIVITY #1

What linear system represents this information?

a) The student council president contacted the two major hotels in town to compare prices for the Pineview Junior High School graduation dinner/dance. The See-Breeze Inn charged a flat fee of $64 plus $13 per guest. The Monarch Motel charged $165 plus $23 per guest. What is the number of guests for which the cost is the same in each hotel?

Let c represent the cost and g represent the number of guests.

b) You rented a car for 5 days, drove 34 km and was charged $166.80. Two weeks later he rented the same car for 10 days, drive 87 km and was charged $337.40. What was the charge per day and per kilometre?

Let x represent the charge per day and y represent the charge per kilometre.

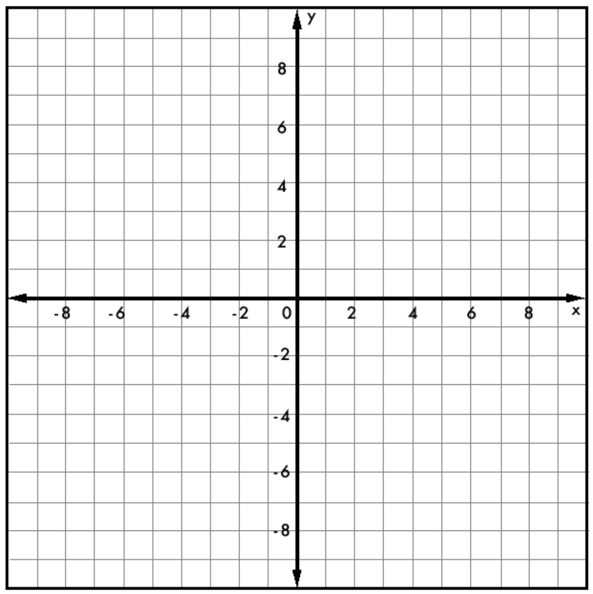
c) Juan invested $640, part at 1.75% per annum and the remainder at 4.25% per annum. After one year the total interest earned was $20.25. How much money did he invest at each rate?

Let a represent amount of money invested at 1.75%, and let b represent amount of money invested at 4.25%

ACTIVITY #2

Solve this system by the graphing method.

$$\begin{cases} -9x+3y=0 \\ 5x-3y=12 \end{cases}$$



ACTIVITY #3

1. Solve this system using substitution.

$$\begin{cases} 5d - 5x = 55 \\ -18d - 3x = 135 \end{cases}$$

2. Solve this system using elimination.

$$\begin{cases} b - 2g = -11 \\ -4b + g = -5 \end{cases}$$

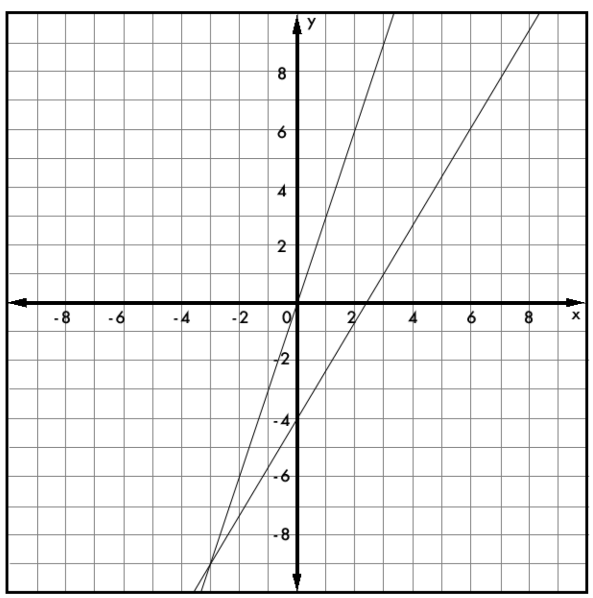
ACTIVITY #1 – SOLUTIONS

a) $\begin{cases} c = 13g + 64 \\ c = 23g + 165 \end{cases}$

b) $\begin{cases} 166.80 = 5x + 34y \\ 377.40 = 10x + 87y \end{cases}$

c) $\begin{cases} a + b = 640 \\ 0.0175a + 0.0425b = 20.25 \end{cases}$

ACTIVITY #2 – SOLUTIONS



ACTIVITY #3 – SOLUTIONS

1.

$$\begin{cases} 5d - 5x = -55 \\ -18d - 3x = 135 \end{cases}$$

$$\begin{aligned} -18d - 3x &= 135 \\ -3x &= 18d + 135 \\ x &= -6d - 45 \end{aligned}$$

$$\begin{aligned} 5d - 5x &= -55 \\ 5d - 5(-6d - 45) &= -55 \\ 5d + 30d + 225 &= -55 \\ 35d &= -55 - 225 \\ 35d &= -280 \\ d &= -8 \end{aligned}$$

$$\begin{aligned} x &= -6d - 45 \\ x &= -6(-8) - 45 \\ x &= 3 \end{aligned}$$

$\therefore$ The solution to the system is $d = -8$ and $x = 3$

2.

$$\begin{cases} 4b - 8g = -44 \\ -4b + g = -5 \end{cases}$$

$$\begin{aligned} -7g &= -49 \\ g &= 7 \end{aligned}$$

$$\begin{aligned} -4b + g &= -5 \\ -4b + (7) &= -5 \\ -4b &= -5 - 7 \\ -4b &= -12 \\ b &= 3 \end{aligned}$$

$\therefore$ The solution to the system is b = 3 and g = 7

FACTORING

Factoring skills are necessary to simplify (reduce) rational polynomial expressions and products of these same expressions.

Thus, the expression

$$\frac{x^2 + 2x - 15}{x^2 + 4x - 21} \quad \text{x} \quad \frac{x^2 + 3x - 28}{x^2 - 25} \quad \div \quad \frac{x^2 + x - 20}{x^2 - 5x}$$

after factoring and reducing becomes $\frac{x}{x+5}$. The identification of restricted values of the variable also needs attention: in this case $x \neq -7, -5$, 0, 3, 4 or 5, since any of these values would result in a denominator of 0 and therefore, an undefined value for the original expression.

Factoring Complex Trinomials of the Form ax^2 + bx + c

Most students do not have great difficulty in factoring simple trinomial quadratic expressions like $x^2 - 2x - 15$, but have more difficulty with those expressions of $ax^2 + bx + c$, for which both a and c are compound numbers (e.g. $12x^2 + 19x - 10$). **Provided that the roots of these expressions are rational**, the following method may be useful.

Using the example, $12x^2 + 19x - 10$, note that the product P of a and c is –120. Various pairs of factors of –120 are sought until the sum of the pairs is +19. This is best accomplished by using a table of values headed by "Product of Factors" and "Sum of Factors".

Note that since P < 0, the factors have opposite signs, and since 19 > 0, the positive factor must be numerically larger than the negative factor. Begin by entering the most apparent pair with this property e.g. (120) (–1). The next simplest factors are (60) (–2). Progressively, the next integral factors of –120 are –3, –4, –5, –6, –8, –10. These are utilized to complete the table. Thus, the required factors of –120 are (24) and (–5).

P = –120	**S = 19**
(120) (–1)	
(60) (–2)	
(40) (–3)	
(30) (–4)	
(24) (–5)	19
(20) (–6)	
(15) (–8)	
(12) (–10)	

Once the proper pair of factors of P has been found, form two factors of the trinomial expression, each of which begins with the term ax:

$(12x+24)(12x-19)$ and compensate for the "extra 12" by dividing by 12.

$$\frac{(12x+24)(12x-19)}{12}$$

The two factors of P are now introduced to complete the basic factored form.

$$\frac{(12x+24)(12x-5)}{12}$$

The expression is reduced.

$$\frac{12(x+2)(12x-5)}{12} = (x+2)(12x-5)$$

Expansion of these factors will yield $12x^2 + 19x - 10$, the original expression.

Example

Factor $8x^2 - 2x - 15$.

P = (8) (-15)
= -120
S < 0

Since P < 0 and S < 0, the factors have opposite signs and the negative factor must be numerically larger than the positive factor.

Factors of P **P = –120**	**Sum** **S = –2**
(–120) (1)	–119
(–60) (2)	–58
(–40) (3)	–37
(–30) (4)	–26
(–24) (5)	–19
(–20) (6)	–14
(–15) (8)	–7
(–12) (10)	–2

$$\frac{(8x-12)(8x+10)}{8}$$

$$= \frac{4(2x-3)(2)(4x+5)}{8}$$

$$= (2x-3)(4x+5)$$

Factoring Quadratics by Completing the Square

It is frequently necessary to change the format of a quadratic expression in order to fit a specific format, For instance, it may be necessary to change the form of $y = x^2 + 6x + 8$ into the form $y = \left(x+3\right)^2 - 1$ which represents a parabola with vertex (–3, –1).

$$y = x^2 + 6x + 8$$
$$y = x^2 + 6x + 9 - 9 + 8$$
$$y = \left(x+3\right)^2 - 1$$

This format may now be factored by the difference of squares to arrive at:

$$y = \left(x+3-1\right)\left(x+3+1\right)$$
$$= \left(x+2\right)\left(x+4\right)$$

Indeed, this method may be utilized to develop the quadratic formula for roots of a quadratic expression. This method will be demonstrated for one specific case and for the general case.

Example I

Factor $2x^2 + 5x - 7$ by the method of completing the square. Let the expression = w.

$$w = 2x^2 + 5x - 7$$

$$\frac{w}{2} = x^2 + \frac{5}{2}x - \frac{7}{2}$$

$$= \quad x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 - \frac{7}{2}$$

$$= \quad \left[x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2\right] - \left(\frac{5}{4}\right)^2 - \frac{7}{2}$$

$$= \quad \left(x + \frac{5}{4}\right)^2 - \frac{25}{16} - \frac{56}{16}$$

$$= \quad \left(x + \frac{5}{4}\right)^2 - \frac{81}{16}$$

$$= \quad \left(x + \frac{5}{4}\right)^2 - \left(\frac{9}{4}\right)^2$$

$$= \quad \left(x + \frac{5}{4} - \frac{9}{4}\right)\left(x + \frac{5}{4} + \frac{9}{4}\right)$$

$$\therefore \frac{w}{2} = (x-1)\left(x + \frac{7}{2}\right)$$

$$\therefore w = 2(x-1)\left(x + \frac{7}{2}\right)$$

$$= (x-1)(2x+7)$$

By extension, the roots are $x = 1$ and $-\frac{7}{2}$

Example II

Factor $ax^2 + bx + c$

Let $= ax^2 + bx + c$

$$\frac{w}{a} = x^2 + \frac{b}{a}x + \frac{c}{a}$$

$$= \quad x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}$$

$$= \quad \left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right] - \left[\frac{b^2}{4a^2} - \frac{4ac}{4a^2}\right]$$

$$= \quad \left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2}$$

$$= \quad \left(x + \frac{b}{2a}\right)^2 - \frac{\sqrt{b^2 - 4ac}^2}{2a}$$

$$= \quad \left(x + \frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}\right)$$

$$= \quad \left(x + \frac{b - \sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b + \sqrt{b^2 - 4ac}}{2a}\right)$$

$$\therefore W = a\left(x + \frac{b - \sqrt{b^2 - 4ac}}{2a}\right)\left(x + \frac{b + \sqrt{b^2 - 4ac}}{2a}\right)$$

Which of course yields roots $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Activity #1

Factor completely.

1.	$3x^4 - 6x^3 + 4x^2$	(GCF)
2.	$p(p-3) + 5(p-3)$	(GCF)
3.	$2n(n+5) - 3(n+5)$	(GCF)
4.	$8r(2r+9) + 5(2r+9)$	(GCF)
5.	$8x^2 + 6x - 20x - 15$	(Grouping; GCF)
6.	$15 - 6x - 20x + 8x^2$	(Grouping; GCF)
7.	$q^2 - 49$	(Difference of Squares)
8.	$16l^2 - 81$	(Difference of Squares)
9.	$4w^2 - 100t^2$	(GCF; Difference of Squares)
10.	$r^2 - 2r + 1 - t^2$	(Difference of Squares)
11.	$4m^2 + 16m + 16 - 25t^2$	(Difference of Squares)
12.	$m^2 - n^2 + 2n - 1$	(Difference of Squares)
13.	$g^2 - m^2 - 12m - 36$	(Difference of Squares)
14.	$d^2 - 6d + 9 - f^2 + 4f - 4$	(Difference of Squares)
15.	$3p^2 - 3q^2 - 36q - 108$	(GCF; Difference of Squares)

Activity #1 - Solutions

1. $x^2\left(3x^2-6x+4\right)$

2. $(p-3)(p+5)$

3. $(n+5)(2n-3)$

4. $(2r+9)(8r+5)$

5. $2x(4x+3)-5(4x+3);(4x+3)(2x-5)$

6. $3(5-2x)-4x(5-2x);(5-2x)(3-4x)$

7. $(q-7)(q+7)$

8. $(4l-9)(4l+9)$

9. $4\left(w^2-25t^2\right);4(w-5t)(w+5t)$

10. $(r-1)^2-t^2;(r-1-t)(r-1+t)$

11. $4\left(m^2+4m+4\right)-25t^2;4(m+2)^2-25t^2;\left[2(m+2)-5t\right]\left[2(m+2)+5t\right];(2m+4-5t)(2m+4+5t)$

12. $m^2-(n-1)^2;\left[m-(n-1)\right];\left[m+(n-1)\right];(m-n+1)(m+n-1)$

13. $g^2-(m+6)^2;\left[g-(m+6)\right]\left[g+(m+6)\right];(g-m-6)(g+m+6)$

14. $(d-3)^2-(f-2)^2;\left[(d-3)-(f-2)\right]\left[(d-3)+(f-2)\right];(d-f-1)(d+f-5)$

15. $3\left[p^2-\left(q^2+12q+36\right)\right];3\left[p-(q+6)\right]\left[p+(q+6)\right];3(p-q-6)(p+q+6)$

ACTIVITY #2

Factor completely.

1. $x^2 + 2x - 8$
2. $x^2 - 2x - 24$
3. $p^2 - 2p - 3$
4. $n^2 + 5n - 6$
5. $2g^2 - 3g - 5$
6. $3g^2 + 2g - 5$
7. $7q^2 - 2q - 5$
8. $7q^2 + 12q + 5$
9. $7q^2 - 12q + 5$
10. $4l^2 - 3l - 1$
11. $v^2 - 3v + 2$
12. $2r^2 - 3r + 1$
13. $2p^2 + 10p - 12$
14. $5z^2 + 10z - 40$
15. $4x^2 - 2x - 6$

Activity #2 - Solutions

1. $(x + 4)(x - 2)$
2. $(x - 6)(x + 4)$
3. $(p - 3)(p + 1)$
4. $(n + 6)(n - 1)$
5. $(2g - 5)(q + 1)$
6. $(3g + 5)(q - 1)$
7. $(7g + 5)(q - 1)$
8. $(7g + 5)(q + 1)$
9. $(7g - 5)(q - 1)$
10. $(4l + 1)(l - 1)$
11. $(v - 2)(v - 1)$
12. $(2r - 1)(r - 1)$
13. $2(p + 6)(p - 1)$
14. $5(z + 4)(z - 2)$
15. $2(2x - 3)(x + 1)$

ACTIVITY #3

Factor completely. Fill in the blanks.

1. $6x^2 - 43x - 15$ P = ☐ S = ☐

Factors	Sum
(–90) (1)	–89
( )(2)	☐
( )(3)	☐
( )(5)	☐
( )(6)	☐
( )(10)	☐

$$\frac{(6x \quad)(6x \quad)}{6}$$

= ( ) ( )

2. $5x^2 + 43x - 18$ P = ☐ S = ☐

Factors	Sum
( )(–1)	☐
( )( )	☐
( )( )	☐
( )( )	☐
( )( )	☐
( )( )	☐

$$\frac{(5x \quad)(5x \quad)}{5}$$

= ( ) ( )

ACTIVITY #3 - Solutions

1. $6x^2 - 43x - 15$ P = –**90** S = –**43**

Factors	Sum
(–90)(1)	–89
(–**45**)(2)	**–43**
(–**30**)(3)	**–27**
(–**18**)(5)	**–13**
(–**15**)(6)	**–9**
(–**9**)(10)	**1**

$$\frac{(6x - \mathbf{45})(6x + \mathbf{2})}{6}$$

$$= (2x - 15)(3x + 1)$$

2. $5x^2 + 43x - 18$ P = –**90** S = **43**

Factors	Sum
(90)(–1)	89
(45)(–2)	43
(30)(–3)	27
(18)(–5)	13
(15)(–6)	9
(9)(–10)	1

$$\frac{(5x + 45)(5x - 2)}{5}$$

$$= (x + 9)(5x - 2)$$

QUADRATIC EQUATIONS

ACTIVITY #1

Solve the equations using factoring. Round any necessary answers to one decimal place.

1. $x^2 - 10x + 24 = 0$

2. $24x^2 + 86x + 77 = 0$

3. $x^2 - 4x - 12 = 0$

4. $x^2 + 3x - 40 = 0$

5. $x^2 - 9x + 14 = 0$

6. $x - 11x + 30 = 0$

7. $96x^2 + 110x + 25 = 0$

8. $192x^2 + 308x + 121 = 0$

9. $96x^2 + 188x + 77 = 0$

10. $128x^2 + 248x + 65 = 0$

ACTIVITY #2

Use the quadratic formula to solve each equation. Round the final $x-$values to 2 decimal places.

1. $7x^2 - 8x + 1 = 0$

2. $-4.8x^2 - 94.3x + 93.9 = 0$

3. $8x^2 - 9x + 2 = 0$

4. $-4x^2 - 91.8x + 98.5 = 0$

5. $-5.7x^2 - 96x + 93.4 = 0$

ACTIVITY #3

Determine the approximate value of the discriminant of each quadratic equation. Round to 2 decimal places.

1. $-7.6x^2 - 15.9x - 95.2 = 0$

2. $-8.5x^2 - 17.9x - 51.8 = 0$

3. $-5.1x^2 - 2.5x - 37.7 = 0$

4. $-8.1x^2 - 25.6x - 52.7 = 0$

5. $-6.1x^2 - 25.6x - 52.7 = 0$

ACTIVITY #1 – SOLUTIONS

1. $(x-4)(x-6)=0$
   $x=4$ or $x=6$

2. $(6x+11)\ (4x+7)=0$
   $x\approx -1.8$

3. $(x-6)(x+2)=0$
   $x=6$ or $x=-2$

4. $(x-5)(x+8)=0$
   $x=5$ or $x=-8$

5. $(x-7)(x-2)=0$
   $x=7$ or $x=2$

6. $(x-6)(x-5)=0$
   $x=6$ or $x=5$

7. $\left(6x+5\right)\left(16x+5\right)=0$
   $x\approx -0.8$ or $x\approx -0.3$

8. $\left(12x+11\right)\left(16x+11\right)=0$
   $x\approx -0.9$ or $x\approx -0.7$

9. $\left(8x+11\right)\left(12x+7\right)=0$
   $x=-1.4$ or $x\approx -0.6$

10. $\left(8x+13\right)\left(16x+5\right)=0$
    $x=-1.6$ or $x\approx -0.3$

ACTIVITY #2 – SOLUTIONS

1.

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(7)(1)}}{2(7)}$$

$$x = \frac{8 \pm \sqrt{36}}{14}$$

$$x = 1 \text{ or } x \approx 0.14$$

2.

$$x = \frac{94.3 \pm \sqrt{(-94.3)^2 - 4(-4.8)(93.9)}}{2(-4.8)}$$

$$x = \frac{94.3 \pm \sqrt{10,695.37}}{-9.6}$$

$$x \approx -20.60 \text{ or } x \approx 0.95$$

3.

$$x = \frac{9 \pm \sqrt{(-9)^2 - 4(8)(2)}}{2(8)}$$

$$x = \frac{9 \pm \sqrt{17}}{16}$$

$$x \approx 0.82 \text{ or } x \approx 0.30$$

4.

$$x = \frac{91.8 \pm \sqrt{(-91.8)^2 - 4(-4)(98.5)}}{2(-4)}$$

$$x = \frac{91.8 \pm \sqrt{10,003.24}}{-8}$$

$$x \approx -23.98 \text{ or } x \approx 1.03$$

5.

$$x = \frac{96\sqrt{(-96)^2 - 4(-5.7)(93.4)}}{2(-5.7)}$$

$$x = \frac{96 \pm \sqrt{11,345.52}}{-11.4}$$

$$x \approx -17.76 \text{ or } x \approx 0.92$$

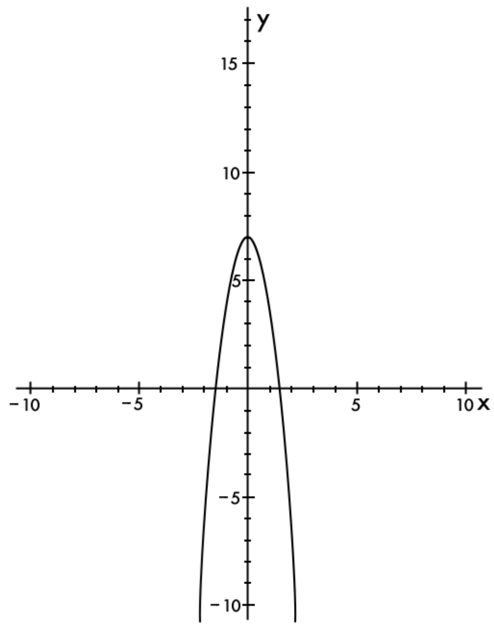
ACTIVITY #3 – SOLUTIONS

1. -2641.27
2. -1440.79
3. -762.83
4. –1052.12
5. -630.52

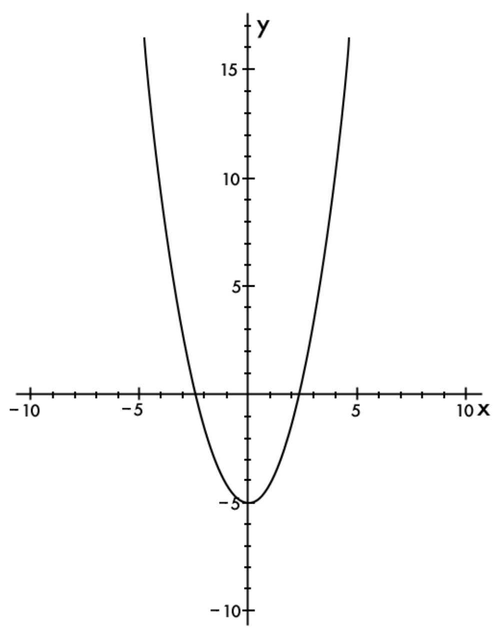
QUADRATIC FUNCTIONS

ACTIVITY #1

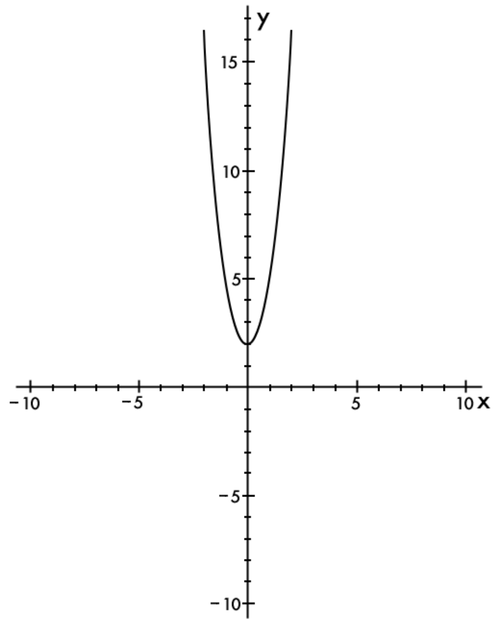
1. What are values for a and c for the quadratic function $y = ax^2 + c$ to match the graph shown?



2. What is the value of c for the quadratic function $y = x^2 + c$ to match the graph shown?



3. What are values for a and c for the quadratic function $y = ax^2 + c$ to match the graph shown?



ACTIVITY #2

1. Given an equation that models a company's profit/loss vs. the number of items sold, what are realistic values for the number of items variable?

   a) Integers

   b) Positive integers

   c) Non-negative integers

   d) Positive integers less that 1000

2. A rock rolls off of a cliff that is 94 m in height. In an equation that models the height of the rock as it falls, what are realistic values for the time variable?

   a) -100 to 100 seconds

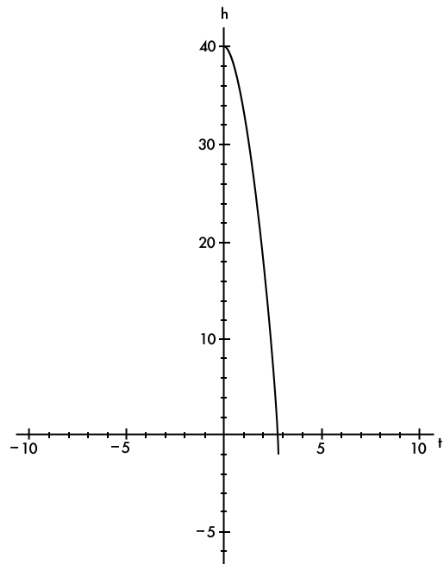
   b) 0 to 100 seconds

   c) -100 to 0 seconds

   d) 100 to 200 seconds

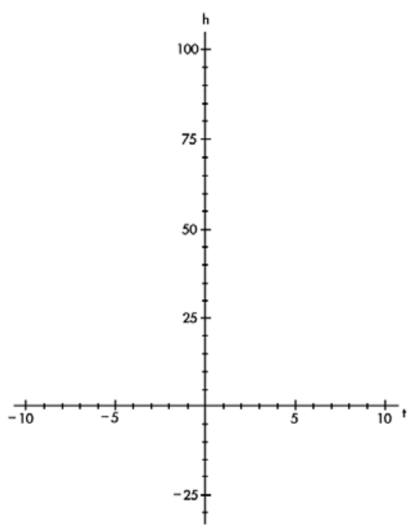
ACTIVITY #3

1. This graph represents the height of a ball released from a height of 40 m as a function of time. How long does it take for the ball to reach the ground.

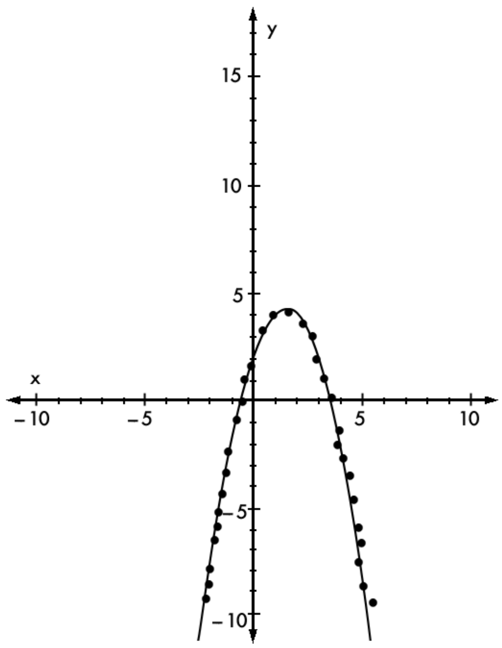


2. A ball is released from a height of 100 m on planet XYZ. It takes 1.7 seconds to reach the ground. Create a graph showing the height of the ball as a function of time.

$$y = -7x^2 + 100$$



3. Predict the y-value when x is 8 for the following scatter plot and curve of best fit. Round to one decimal place.



ACTIVITY #1 – SOLUTIONS

1. a = -4
   c = 7

2. c = -5

3. a = 4
   c = 2

ACTIVITY #2 – SOLUTIONS

1. d) positive integers less than 1000.

   Since a company cannot make a negative number of items, there cannot be negative values. A company cannot make an infinite number of items, so there must be some upper limit.

2. b) 0 to 100 seconds

   The time variable cannot be negative, since the equation only represents the rock's height as it falls. The rock would likely hit the ground within 100 seconds.

ACTIVITY #3 – SOLUTIONS

1. 2.9 seconds
   The graph meets the time axis at 2.9 seconds. Therefore, it takes the ball 2.9 seconds to reach the ground.

2. see question 2 for answer.

3. -38

INEQUALITIES

ACTIVITY #1

What are the solutions to these inequalities?

(a) $4x+2<-2x+7$

(b) $6(-9x+6)+9<8x-6$

(c) $-9(3x-1)-1\le -8(-2x-2)$

ACTIVITY #2

Carl has a budget of $4400 for a trip to Europe. The cost for transportation is fixed at $800 plus an additional cost of $42 per day. If he stays within his budget, how long can he stay in Europe?

ACTIVITY #3

You are designing a triangular garden for a landscaper. The perimetre must be less than 23 m. If two of the sides have lengths of 6 m and 7 m, what are the possible lengths for the third side?

ACTIVITY #1 – Solutions

(a)
$$4x+2 < -2x+7$$
$$4x+2+2x < -2x+7+2x$$
$$6x+2 < 7$$
$$6x+2+(-2) < 7+(-2)$$
$$6x < 5$$
$$6x \div 6 < 5 \div 6$$
$$x < \frac{5}{6}$$

(b)
$$6(-9x+6)+9 < 8x-6$$
$$-54x+36+9 < 8x-6$$
$$-54x+45 < 8x-6$$
$$-54x+45-8x < 8x-6-8x$$
$$-62x+45 < -6$$
$$-62x+45-45 < -6-45$$
$$-62x < -51$$
$$-62x \div -62 > -51 \div -62$$
$$x > \frac{51}{62}$$

(c)
$$-9(3x-1)-1 \le -8(-2x-2)$$
$$-27x+9-1 \le 16x+16$$
$$-27x+8 \le 16x+16$$
$$-27x+8-16x \le 16x+16-16x$$
$$-43x+8 \le 16$$
$$-43x+8-8 \le 16-8$$
$$-43x \le 8$$
$$\frac{-43x}{-43} \ge \frac{8}{-43}$$
$$x \ge \frac{-8}{43}$$

ACTIVITY #2 – Solutions

Let n = number of days.

$$42n + 800 \leq 4400$$
$$42n + 800 - 800 \leq 4400 - 800$$
$$42n \leq 3600$$
$$42n \div 42 \leq 3600 \div 42$$
$$n \leq \frac{600}{7}$$
$$n \leq 85.71$$

Therefore, Carl can stay in Europe for 85 days.

ACTIVITY #3 – Solutions

Let a, b, and c represent the lengths of each side.

$$a + b + c \leq 23$$
$$a + 6 + 7 \leq 23$$
$$a + 13 \leq 23$$
$$a \leq 10$$

Possible lengths are any length between 0 and 10 m, excluding 0 and including 10.

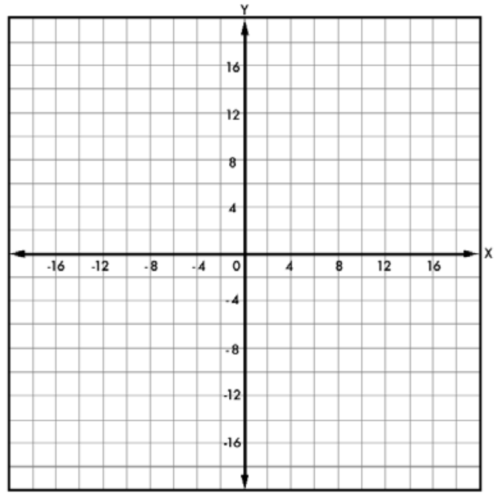
In order for the line segments to form a triangle, the maximum length of the third side is 2 m.

LINEAR INEQUALITIES

ACTIVITY #1

a) Graph this inequality.

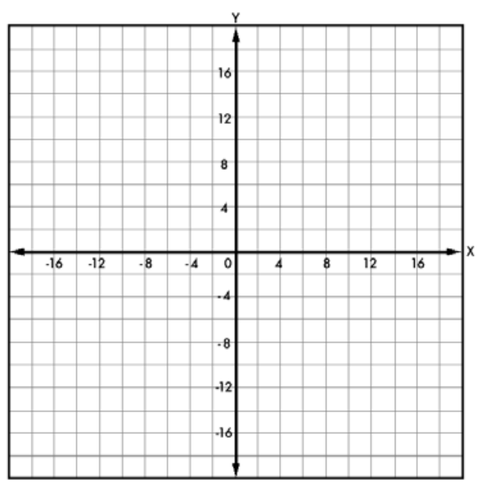
$$y < -22x - 314$$



b) Graph these inequalities.

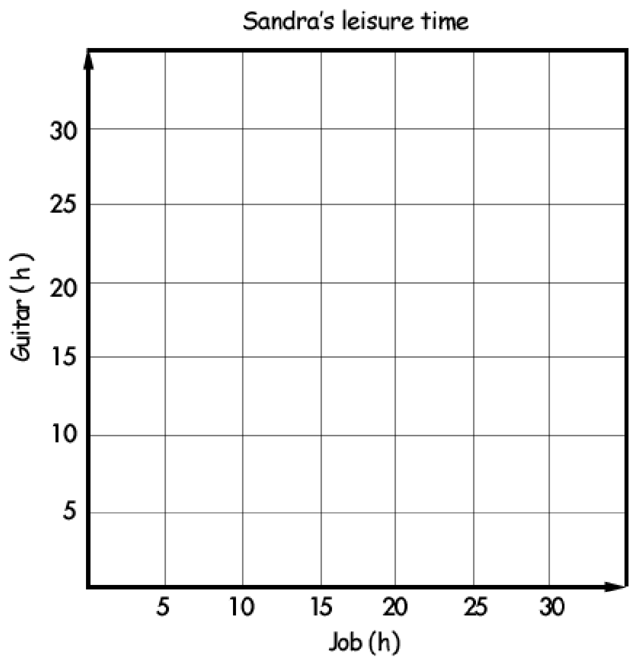
$$y < 2x - 3$$

$$y \leq 9x - 54$$



ACTIVITY #2

Sandra allows herself 5 hours a week to either work at a job or to practice playing the guitar. The rest of her time has to be spent on school work. Sometimes she has less than 5 hours of leisure time because of exams or family obligations. Represent Sandra's situation as a linear inequality on a graph.

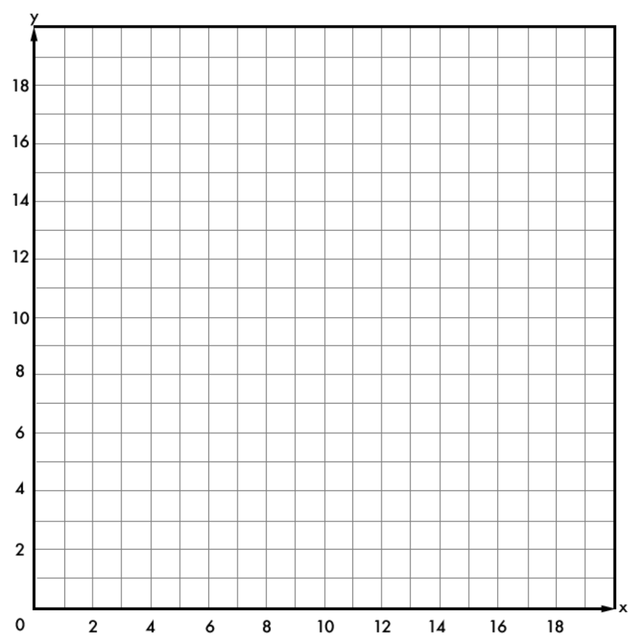


ACTIVITY #3

a) Construct the graph that represents this information.

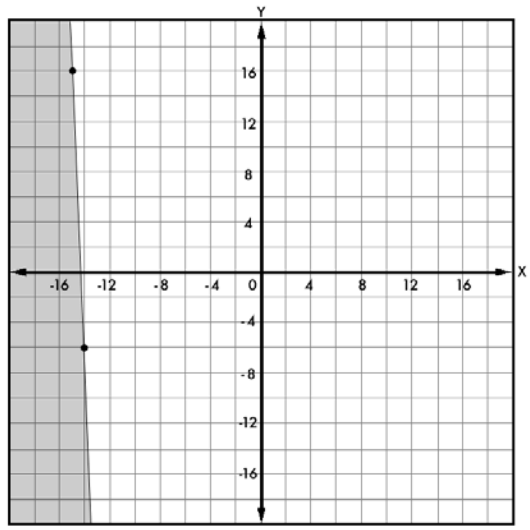
$$\begin{cases} 2(3x+2y) \leq 72 \\ 2(8x+16y) \leq 224 \end{cases}$$

b) Explain how you arrived at the graph.



ACTIVITY #1 – SOLUTIONS

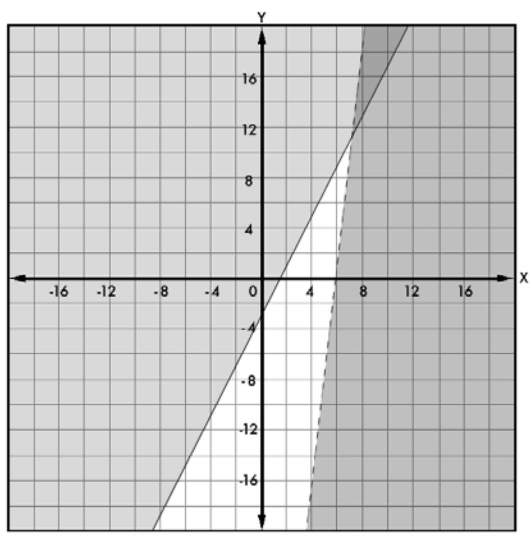
1.



NOTE:
Test if (0, 0) is part of the solution by substituting a value of 0 for both x and y. If the resulting statement is true, shade the side that includes (0, 0), otherwise shade the opposite side.

If the inequality contains an equals sign, then the boundary line is solid; otherwise it is dotted.

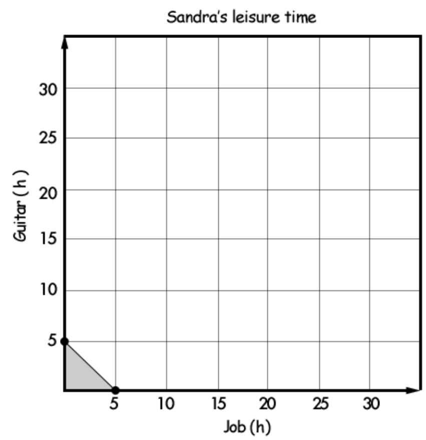
2.



ACTIVITY #2 – SOLUTIONS

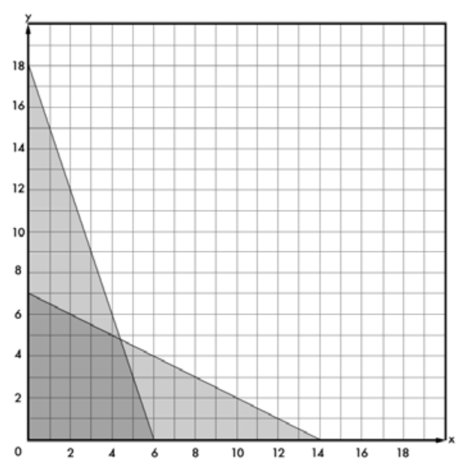
NOTE: $x + y \leq 5$

The lines y = 0 and x = 0 are solid because some weeks Sandra will not be able to work nor play the guitar. All three lines that form the boundaries are solid.



ACTIVITY #3 – SOLUTIONS

a)



FUNCTIONS

ACTIVITY #1

This table shows the charges for bicycle rental for up to a week. An initial deposit is included in the amounts shown.

(a) Write an equation for the function.

(b) Write a set of ordered pairs for the function.

(c) Identify the domain and range values.

Number of days, x	1	2	3	4	5	6	7
Rental cost, y	15	20	25	30	35	40	45

ACTIVITY #2

For each relation, describe the domain and range. Determine whether the relation is a function.

(a) $\{(0,3),\ (0,4),\ (4,0),\ (3,0)\}$

(b) $\{(-1,-2),\ (1,-2),\ (3,-1),\ (-3,-2),\ (0,4)\}$

(c) $\{(9,9),\ (3,3),\ (4,4),\ (5,5),\ (12,12)\}$

ACTIVITY #1 - Solutions

(a) $y = 5x + 10$

(b) $\{(1,15), (2,20), (3,25), (4,30), (5,35), (6,40), (7,45)\}$

(c) Domain: {1, 2, 3, 4, 5, 6, 7}
Range: {15, 20, 25, 30, 35, 40, 45}

ACTIVITY #2 - Solutions

(a) Domain: {0, 4}
Range: {0, 3, 4} Not a function

(b) Domain: {-3, -1, 0, 1, 3}
Range: {-2, -1, 4} A function

(c) Domain: {3, 4, 5, 9, 12}
Range: {3, 4, 5, 9, 12} A function

RATIONAL EXPRESSIONS

ACTIVITY #1

1. Simplify the expression. $\dfrac{16x-8}{24x}$

2. Which expression is an expression equivalent to $\dfrac{10e^5i^4v^3}{4ei^2v^6}$?

   (a) $\dfrac{2v^3}{5e^4i^2},\ i\neq 0, v\neq 0$

   (b) $\dfrac{5e^4i^2}{2v^3},\ e\neq 0, i\neq 0, v\neq 0$

   (c) $\dfrac{10e^4i^2}{2v^3},\ e\neq 0,\ i\neq 0,\ v\neq 0$

3. Simplify

   $$\frac{1(3+5x)}{1\left(7+2y\right)}\div\frac{1(3+5x)}{-z(7+2y)}$$

4. Which expression is an expression equivalent to $\dfrac{a^2-64}{7a+56}$?

   (a) $\dfrac{a-8}{7a+56},\ a\neq -8$

   (b) $\dfrac{a^2-64}{7},\ a\neq -8$

   (c) $\dfrac{a-8}{7},\ a\neq -8$

ACTIVITY #2

Which is a better value?

1.5 l bottle for $1.29
or
A case of 24 x 355 ml cans for $5.89

ACTIVITY #3

Your batting statistic is 12 hits in 28 attempts at bat.

(a) How many attempts at bat would you need for 200 hits?

(b) What assumption did you make?

ACTIVITY #1 - **Solutions**

1. $$\frac{16x-8}{24x}=\frac{8(2x-1)}{24x}$$

$$=\frac{2x-1}{3x}$$

Restriction: The denominator of the original expression (24x) cannot be zero.
Therefore, $x \neq 0$.

2. Answer (b)

3. $$\frac{1(3+5x)}{1(7+2y)} \div \frac{1(3+5x)}{-z(7+2y)}=\frac{1(3+5x)}{1(7+2y)} \bullet \frac{z(7+2y)}{1(3+5x)}$$

$$=\frac{1}{1} \bullet \frac{-z}{1}$$

$$=-z$$

4. Answer (c)

ACTIVITY #2 - **Solutions**

Bottle:		1.5 l	costs \$1.29
		1 l	costs \$1.29 / 1.51
			\$0.85
Cans :	24 x	355 ml	= \$8 520 ml
			= \$8.52 l
		8.52 l	costs \$5.99
		1 l	costs 8.52
			\$0.70

Therefore, the better value is the case of cans

ACTIVITY #3 - Solutions

(a) 12 hits in 28 attempts

1 hit in $\frac{28}{12}$ attempts

200 hits requires $\frac{28}{12} \times 200$ or 467 attempts

$$\begin{aligned} 12:28 &= 1:\frac{28}{12} \\ &= 200:\frac{28}{12}\times 200 \\ &= 200:466\frac{2}{3} \end{aligned}$$

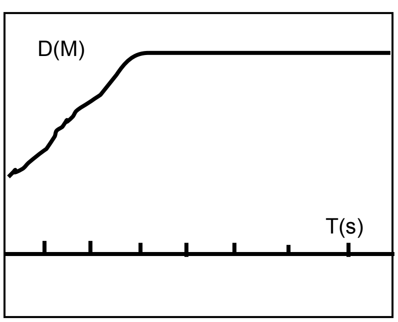
(b) Assumption:
You maintain the same ratio of hits to attempts.

Performance Tasks

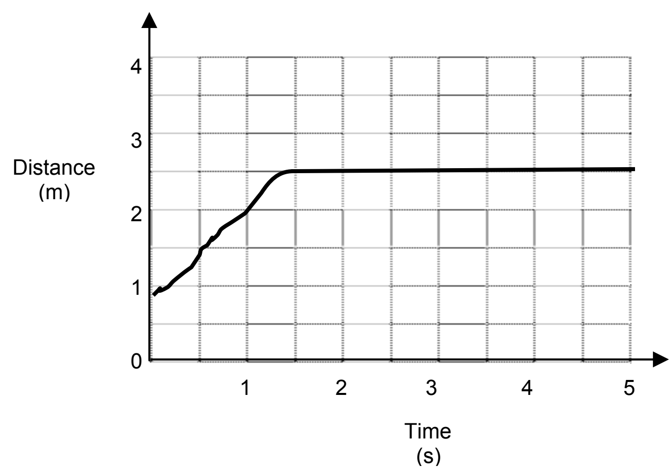
TASK #1: DESCRIBING A GRAPH

1. A person walked away from a motion detector.

Below is a screen captured from the graphing calculator and a graph representing his walk.



The Walk



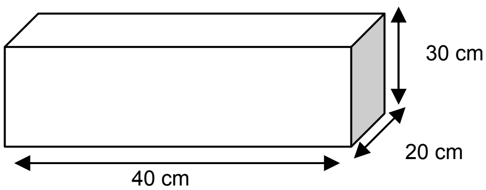
Using this information, describe the person's walk in mathematical language.

TASK #2: AQUARIUM ANALYSIS

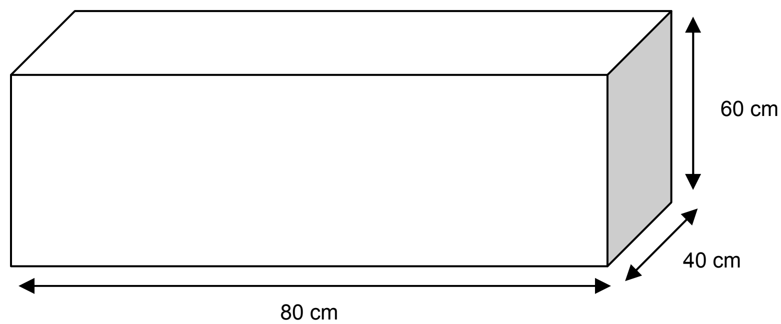
Fishmart sells aquariums in the shape of rectangular prisms. The aquariums are available in two sizes, small and large, with dimensions as shown. Each aquarium has glass sides and bottom, but no top.

NOTE: These aquariums are NOT drawn to scale.

SMALL



LARGE



(a) Calculate the volume of each aquarium

Small	Large

(b) Calculate the total outside surface area of each aquarium.

Small	Large

(d) The cost of materials required to build the aquariums is $0.003 cm^2 of surface area. Determine the cost of materials required to build each aquarium. Show your work.

Small	Large

(d) The large aquarium sells for $150 and the small for $55.

Do the selling prices of the aquariums seem appropriate according to your calculations? **Give reasons for your answer.**

____

____

____

____

(e) A buyer thinks the large aquarium should only cost two times as much as the small aquarium for these reasons:

- The dimensions of the large aquarium are two times bigger than those of the small aquarium.
- It takes two times more material to build the larger aquarium.
- What is the mathematical error in the buyer's reasoning?

TASK #3

STATISTICS IN SPORTS: MEASURES OF CENTRAL TENDENCY

The Task:

You will develop a profile of a current sports player. Select a sport and a player that has been playing for at least five years. Research the statistics for your player and create a presentation that includes the mean, median, mode, range, and graph of the player's statistics.

> **Hint:** Use the Internet to obtain the statistics.

Process:

1. Complete the section of Math Trek High School on Statistical Relationships. You may also wish to access a lesson on measures of central tendency at www.aaamath.com/sta.html

2. Choose a player with at least five years of statistics. You may wish to access NBA.com (basketball), MLB.com (baseball), NHL.com (hockey), WNBA.com (women's basketball).

3. Use your player's statistics to find the measures of central tendency.
   a) For basketball, use points per game (ppg).
   b) For baseball, use batting average (avg).
   c) For hockey, use goals (g).

4. Plot the statistics from the last five years and make a graph. Find the mean, median, mode, and range.

5. Use this data and your graph to predict what your player's statistics will be for the next season.

6. Create your presentation. It may be in poster or electronic format.

TASK #4

RIDING A ROLLER COASTER

Introduction:

You have been charged with finding the three most thrilling roller coaster rides.

Task:

Use Internet resources to study the roller coasters at ten major amusement parks. Based on this data, you will recommend the three most thrilling rides.

Process:

1. Find sites which describe the roller coasters at major amusement parks.
2. Select ten rides to study.
3. Obtain data for at least five variables related to each coaster, such as height, length, and speed.
4. Present data in a table and also in a graph format with each ride graphed in a different colour.
5. Select the three most thrilling rides.
6. Prepare a report. You should explain your definition of most thrilling, and justify why you chose these three rides as most thrilling.

TASK #5

CRITIQUING PROOFS

The aim of this task is to:

- evaluate 'proofs' of given statements and identify which are correct; and
- identify errors in 'proofs.'

Consecutive Addends
Here are three attempts at proving the following statement:

When you add three consecutive numbers, your answer is always a multiple of three.

Look carefully at each attempt. Which is the best 'proof'? Explain your reasoning as fully as possible.

Attempt 1:

1 + 2 + 3 = 6	3 x 2 = 6
2 + 3 + 4 = 9	3 x 3 = 9
3 + 4 + 5 = 12	3 x 4 = 12
4 + 5 + 6 = 15	3 x 5 = 15
5 + 6 + 7 = 18	3 x 6 = 18

And so on. So it must be true.

Attempt 2:

3 + 4 + 5
The two outside numbers (3 and 5) add up to give twice the middle number (4).
So all three numbers add to give three times the middle number. So it must be true

Attempt 3:

Let the numbers be:

n, $n + 1$ and $n + 2$

Since

$$n + n + 1 + n + 2 = 3n + 3 = 3(n + 1)$$

It is clearly true.

The best proof is attempt number ....

This is because ....

TASK #6

International Shopping Spree

Introduction:

You have decided to compare prices for specific products in four different English-speaking countries. You will determine the costs for the same merchandise, convert the prices using current exchange rates, and choose the best buy, excluding shipping costs.

Task:

Using Internet resources for shopping in the United States, Canada, Australia, and Great Britain, you will select 5 items to buy in all four countries. Once you find the local cost, you will convert that price to Canadian dollars and compare the costs.

Resources:

Yahoo Australia Shopping: http://au.shopping.yahoo.com/
Yahoo Canada Shopping: http://ca.shopping.yahoo.com/
Yahoo UK and Ireland Shopping: http://shopping.uk.yahoo.com/
Yahoo US Shopping: http://shopping.yahoo.com/
Exchange Rate Converter: http://www.oanda.com/converter/classic

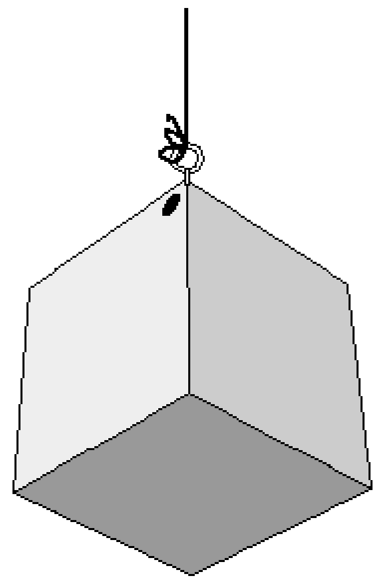
Process:

You will browse the online shopping resources for the United States, Canada, Australia, and Great Britain.

1. Select five items to study.
2. Look up the cost in all four countries for each of the items.
3. Using current exchange rates, convert all prices to Canadian dollars.
4. Make a table and a graph to illustrate results.
5. Write a report describing your results. Consider answers to these questions:
    - (i) Is the cost of each item the same in all countries?
    - (ii) Is there a pattern where all items are more or less in some countries?
    - (iii) Can you find an explanation?

TASK #7

Chocolate Polyhedra



Task Description
Students are asked to visualize the shape formed when a cube half full of chocolate is left to set in different positions. The task is a spatial reasoning one. Students need to visualize and describe the shape.

At the end of the task students are given a part statement of Euler's Formula for any polyhedra and asked to use their work to figure out the complete formula.

Assumed Mathematical Background
Students should have had some experience working with 3-dimensional figures. It is expected that all students will be familiar with the terms face, vertex, and edge.

Core Elements of Performance
The task provides students with the opportunity to:
- visualize 3-D shapes;
- sketch polyhedra;
- use Euler's Formula, (F) + (V) - (E) is equal to a certain number, and to figure out the number.

Circumstances
Grouping: Students work individually.
Materials: A cube will be useful to students who have difficulty tackling this task.
Estimated time: 45 minutes
- Visualize 3-D shapes.
- Sketch polyhedra.
- Explore Euler's Formula.

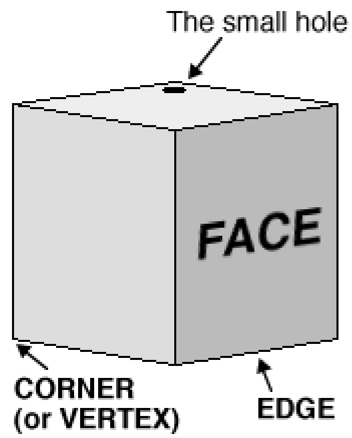
Chocolate Polyhedra

The aim of this assessment is to provide the opportunity for you to:

- show how you can visualize 3-dimensional shapes;
- carry out an investigation.

Imagine that you work in a chocolate factory and that you are responsible for designing interestingly shaped chocolates.

You have several plastic molds in the shape of a cube. They look like the one shown opposite.



Chocolate is poured into each mold through the small hole so that when set the mold is exactly half full.

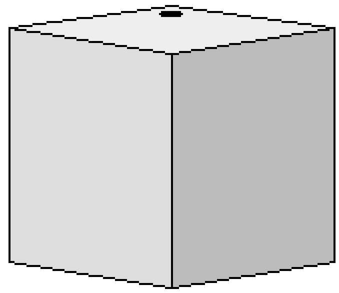
To make different shaped chocolates, the molds are left to set in different positions.

Each of the diagrams on the next page shows the position that the mould was left to set.

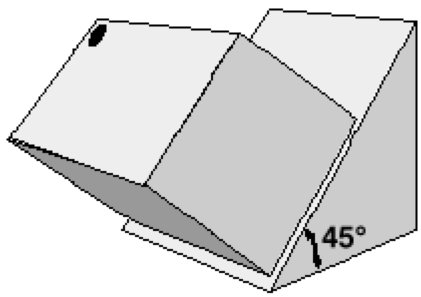
Look at each of the following diagrams.

- Make a sketch of the chocolate piece that is made.
- Record the number of faces that it has.
- Record the number of corners.
- Record the number of edges.
- Describe, as fully as possible, the shape that is made.

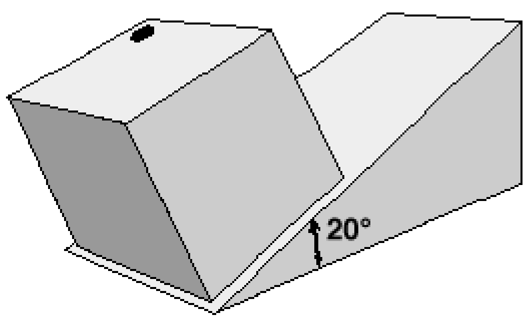
1. The mould sets while resting on one face.



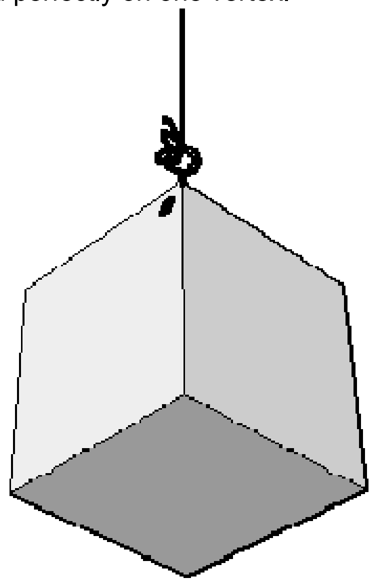
2. The mould sets while balanced perfectly on one edge at an angle of 45 degrees to the horizontal.



3. The mould sets while tilted on one edge at an angle of 20 degrees to the horizontal.



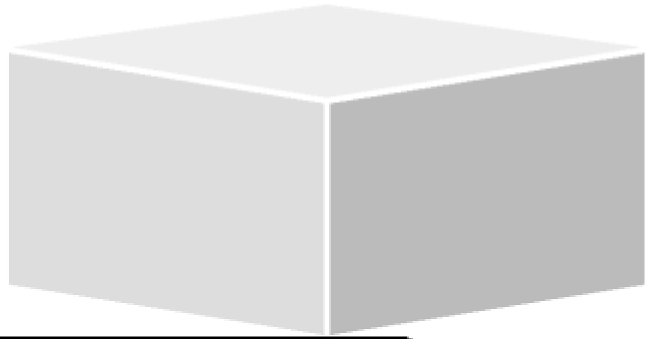
4. The mould sets while balanced perfectly on one vertex.

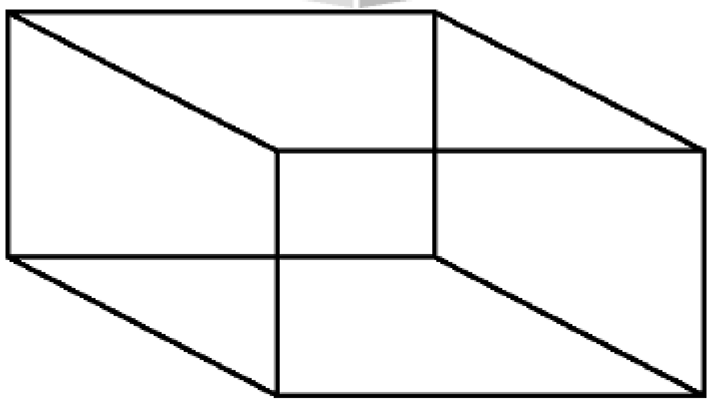


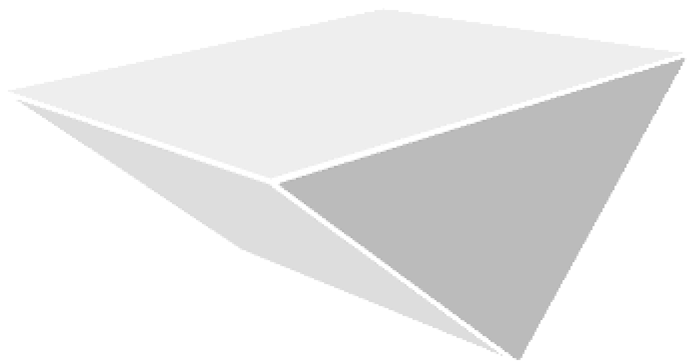
Chocolate Polyhedra: Sample solution

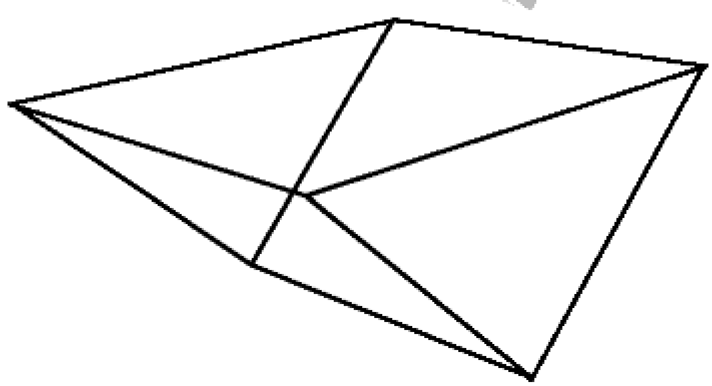
A Sample Solution

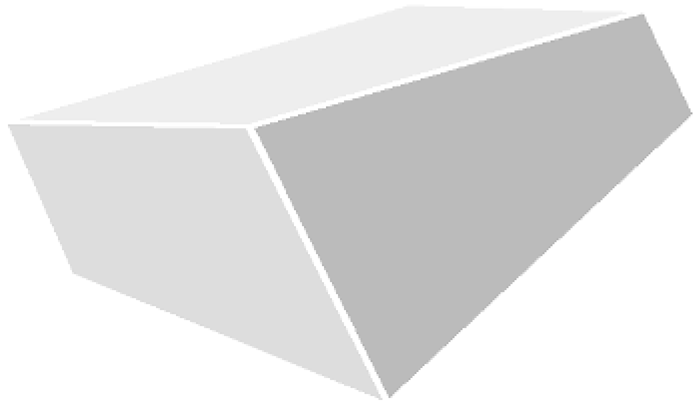
Position	number of faces	number of corners	number of edges
1. resting on a face	6	8	12
2. resting on an edge (45º)	5	6	9
3. resting on an edge (200º)	6	8	12
4. resting on one vertex	7	10	15

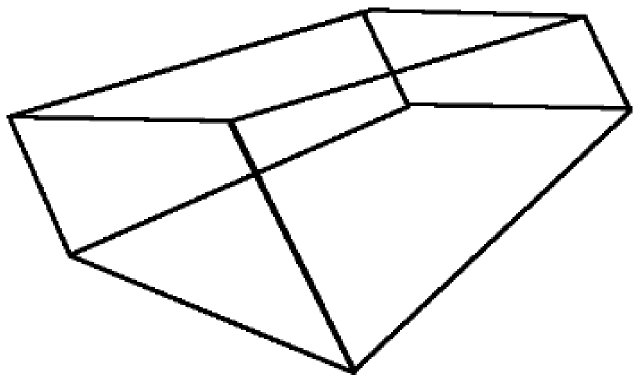


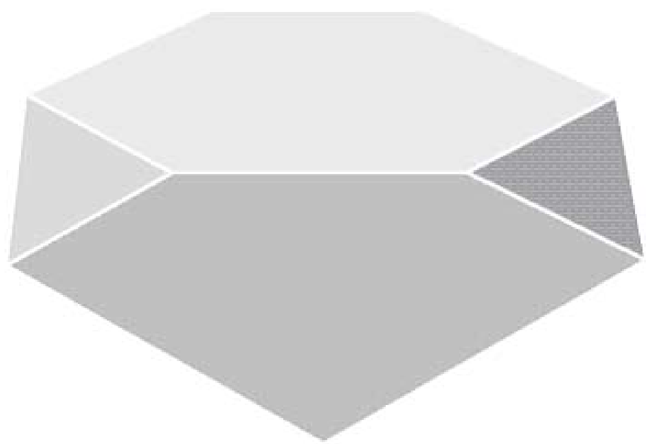


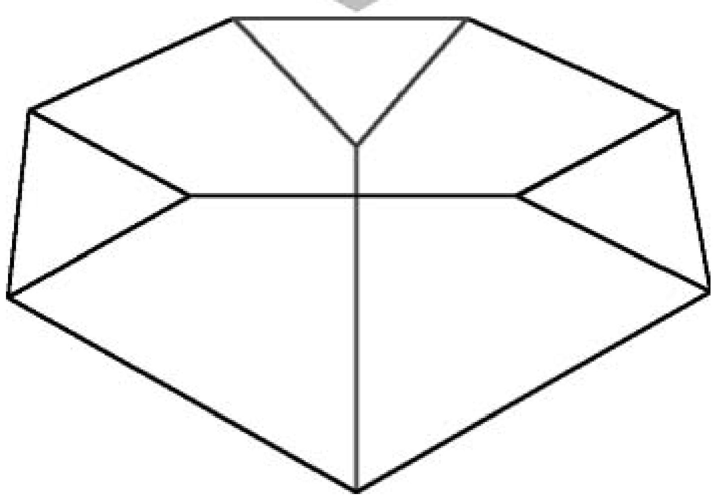












Chocolate Polyhedra: Using this task

Extensions

Euler's Formula says that for any polyhedra:

The number of faces (F) + the number of vertices (V) - the number of edges (E) is equal to a certain number.

Devise and report an investigation of this formula. Find out what that certain number is. You may use this work or any example of polyhedra. Packaging, especially those used for chocolate boxes are a good source. In your report include interesting polyhedra that illustrate Euler's Formula.

For Formal Assessment

Students usually find question 4 challenging. A clear cube that is half full of salt may be necessary to enable students to `see' the correct solution.

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TASK #8

Design a Tent

Task Description

Students are asked to design a tent, making estimates for suitable dimensions. Explain that if students want to, they may find it helpful to make paper models of their experimental designs to check that they work. A supply of scissors and paper should be available if students request them.

Assumed Mathematical Background

Students should have experience of estimation, drawing nets of 3-D objects, using the Pythagorean Theorem and/or the trigonometric ratios.

Core Elements of Performance:

This task provides students with the opportunity to:

- estimate dimensions of an adult that would need to fit into the tent;
- visualize and sketch what the net of a tent looks like;
- calculate measurements and label the sketch;
- apply the Pythagorean Theorem;
- apply the trigonometric ratios.

Circumstances

Grouping: Students work individually or in pairs. Pair work should encourage discussion. Each member of a pair, however, must produce his or her own written solution.

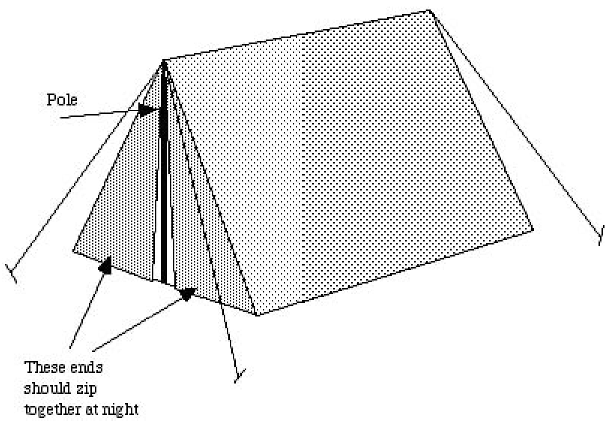
Materials: Each pair of students will need a scientific calculator. Students may request a pair or scissors and some paper for model making.

Estimated time: 45 minutes

- Estimate dimensions.
- Visualize and sketch a net for a tent.
- Calculate measurements using the Pythagorean theorem and/or the trigonometric ratios.

Design a Tent

The aim of this assignment is to:
- estimate dimensions of a person;
- visualize and sketch a net for a tent, showing all the measurements.



Your task is to design a tent like the one in the picture.

Your design must satisfy these conditions:

- It must be big enough for two adults to sleep in (with their baggage).
- It must be big enough for someone to move around in while kneeling down.
- The bottom of the tent will be made from a thick rectangle of plastic.
- The sloping sides and the two ends will be made from a single, large sheet of canvas. (It should be possible to cut the canvas so that the two ends do not need sewing onto the sloping sides. It should be possible to zip up the ends at night.)
- Two vertical tent poles will hold the whole tent up.

1. Estimate the relevant dimensions of a typical adult and write these down.

2. Estimate the dimensions you will need for the rectangular plastic base.

3. Estimate the length of the vertical tent poles you will need.
   Explain how you get these measurements.

4. Draw a sketch to show how you will cut the canvas from a single piece.
   Show all the measurements clearly.
   Calculate any lengths or angles you don't know.
   Explain how you figured out these lengths and angles.

Design a Tent: Sample solution

A Sample Solution

1. The height and width of typical males and females are given in the table below, together with the range within which approximately 90% of the population lie.

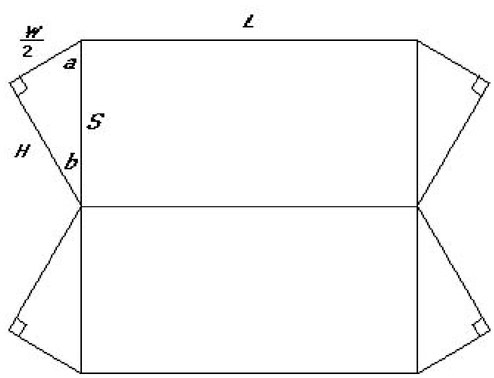
	Average	Range
Males: Height	1740mm / 5' 9"	1625 - 1855mm / 5' 4" - 6' 2"
Males: Width	460mm / 1' 6 "	415 - 510 mm / 1' 4" - 1' 8"
Females: Height	1610mm / 5' 4"	1505 - 1710 mm / 4' 11" - 5' 8"
Females: Width	415mm / 1' 4"	370 - 460 mm / 1' 3" - 1' 6"

Accept any answer that lies between
Height 5' 4" to 6' 2"; Width 1' 4" to 2'; Kneeling height 4' to 4'6".

2. It would thus seem sensible to design the plastic base to be at least 6' 6" in length and at least 4 feet wide. More would be better for baggage.

3. The length of the tent poles has to lie between the above height and kneeling height.

4. In the drawing below, the tent is made to fit a base L units long and W units wide with tent poles of height H units.



The dimension labeled *S* may be found using the Pythagorean theorem.

The angles labeled *a* and *b* may be found using trigonometric ratios.

Design a Tent: Using this task

Introducing the Task

Use the following script:

"This task asks you to design a tent like the one in the picture. Read the conditions carefully. You can use measurements in feet, inches, metres or centimetres - whichever you prefer.

If you need to make a rough model of the tent to help you visualize your ideas, then scissors and paper are available. Don't spend much time making models. You may not need to make a model at all. You will only be scored on your written answers to questions 1 through 4.

Make sure you write down any assumptions you make and show all your work."

While Students Work

Reemphasize the directions in the introduction. Prompt students to state their assumptions and show their work.

Do not prompt them to use the Pythagorean theorem or trigonometric ratios.

Wrap Up

Tents come in a wide variety of designs. You may like to suggest that students sketch out the plans for making a different design (for example where the ridge is sloping) or even an original design of their own.

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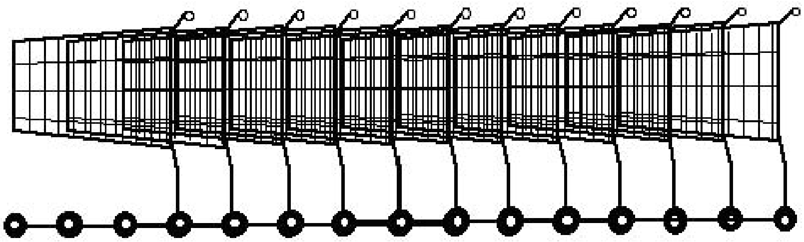
Task # 9

Supermarket Carts



Task Description

This task asks students to analyze a situation where supermarket carts from a supermarket are "nested" inside one another in a row. They are given a scale diagram of nested carts.

The goal is to create a rule for the length of a row of nested carts given the number of carts. Finally, the student is asked to give the rule for and the number of carts given a space S metres long.

- Develop an approach to a relatively unspecified task.
- Model a situation with a discrete linear function.

Assumed Mathematical Background

It is essential that students have experience attempting non-routine problems. Students need to have done work in algebra in which they have had a chance to work with simple linear functions and their applications. It is also essential that students will have had some experience defining variables.

Core Elements of Performance:

- being able to formulate an approach to a task that is not fully specified;
- being able to model a structure symbolically; that is, recognize and generalize the pattern generated by a stack of nested supermarket carts;
- being familiar with linear functions of a discrete variable n (that is, functions such as $y = 25n + 8$ where n is a whole number);
- knowing how to set up and use such a function to model a given situation;
- being able to find the inverse of this function.

Circumstances

Grouping: Students work individually, in pairs, or in groups of four.
Materials: A ruler and pencil is all that is necessary, but some approaches might find a calculator useful.
Estimated time: 45 minutes

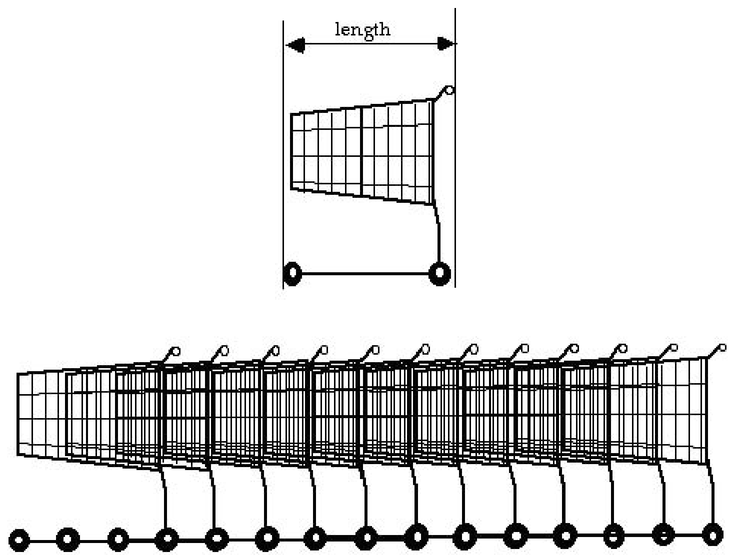
Supermarket Carts

The aim of this assessment is to:

- think mathematically about supermarket carts;
- create a rule that can be used to predict the length of storage space needed, given the number of carts.

The diagram below shows a drawing of a single supermarket cart.
It also shows a drawing of 12 supermarket carts that have been "nested" together.

The drawings are **one twenty-fourth (1/24th) real size**.



1. Create a rule that will tell you the length of storage space (S) needed when all you know is the number of supermarket carts to be stored.
   You will need to show HOW you built your rule; that is, we will need to know what data you drew upon and how you used it.

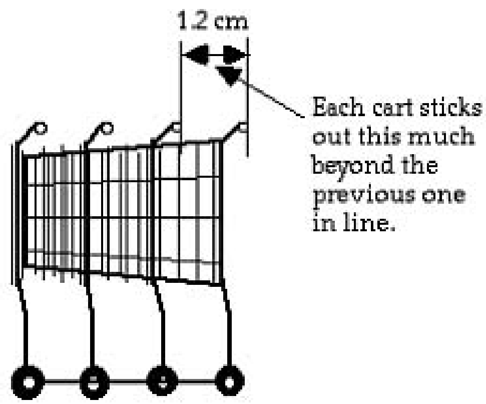
2. Now show how you can figure out the number of carts that can fit in a space S metres long.

Supermarket Carts Sample solution

A Sample Solution

The length on the diagram is about 4 cm. Since the scale of the diagram is given as 1/24th, the length of a full-size supermarket cart is (40) (24) = 96 cm = .96 metres.

When they are "nested", each supermarket cart sticks out beyond the next one in line by about 1.2 cm. (A good technique is to measure 10 carts in a row and divide by 10.) This is a full-size length of about (1.2) (24) = 28.8 cm. = .288 m.



The first cart measures .96 m. Each other cart sticks out .288 m.

Generalizing n carts take up **S = .96 + (.288) (n-1)** metres.

Rearranging for n gives **n =** ((S - .96) / .288) + 1.

Supermarket Carts: Using this task

Using This Task

N.B. When you use this task, do not tell students that they should or may use a ruler. Use this task as an opportunity for students to decide what mathematical tools are needed.

Informal classroom use of this task is good when students are working on linear functions of the form y = mx + b. In this task, the independent variable is a whole number n that counts the number of carts. It may take some practice before students see the "meaning" of the two parametres in terms of the application. To illustrate, consider the formula for the space S in metres taken up by n carts:

$$S = .96 + (.288)(n-1)$$

This is a convenient form to use when computing S from n, but it is not in "slope-intercept" form. This form is

$$S = (.288)(n) + .672$$

The "slope" m is .288 (metres/cart), which is the amount each cart sticks out beyond the previous cart in line. The "y-intercept" b is .672 metres, which is the part of the cart which doesn't stick out.

For Formal Assessment

If you are using this task as part of a formal system of assessment, it should be presented to students with standardized instructions.

Extensions

Supermarket Carts can be extended for instructional purposes by having students do more work with this situation. They can graph the function for S in terms of n, and show how to go from n to S or S to n graphically. The results can be checked by actual measurements on the diagram for n up to 12.

Students can also be asked to come up with their own real world examples illustrating linear functions.

For assessment purposes, other "stacking" or "nesting" situations similar to this can be given for students to analyze. A simple one is stacks of paper cups.

The New Standards task Shoelaces is another application of linear function.

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